MINISTRY OF CULTURE · GOVERNMENT OF INDIA
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NOTES

ON

Practical Astronomy,

BEING PART OF SURVEY COURSE

AT THE

SCHOOL OF MILITARY ENGINEERING,

CHATHAM.

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S.M.E., CHATHAM,
March 11*th*, 1875.

These Notes have been prepared for the use of R.E. Officers going through the Astronomical Course.

The present Edition is in a much more portable form than the Edition existing up to this date. It contains considerably more information, especially as regards the determination of Longitude.

Many of the explanations have been re-written, and additional explanations given where thought necessary.

The revised Edition has been mainly compiled by Captains Fellowes and Livesay, R.E., Assistant Instructors in the Survey Branch.

F. E. PRATT,
MAJOR, R.E.,
Superintendent Survey.

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सत्यमेव जयते
संस्कृति मंत्रालय · भारत सरकार

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Extracts taken from the following Works which are recommended for reference :—

CHAUVENET ASTRONOMY.
LOOMIS' ASTRONOMY.
GODFRAY'S ,,
HERSCHEL'S ,,
GALBRAITH & HAUGHTON'S ASTRONOMY.
MRS. TAYLOR'S TABLES.
NORMAN LOCKYER & "THE HEAVENS."
HARNESS' PAPERS.
MAIN'S PRACTICAL ASTRONOMY.

ASTRONOMICAL COURSE.

The Course consists of purely practical astronomy, such as will enable an officer to ascertain his time, latitude, and longitude. These will be found by the following methods :—

TIME.	By observed transit. By single altitudes of sun or star. By equal ,, ,, same star. By ,, ,, of any two stars.
LATITUDE.	1. By a meridian altitude of sun or star. 2. Circummeridian ,, ,, 3. Single altitude, near meridian, of sun or star. 4. Altitude of Polaris at any time. 5. Altitude of circumpolar star at upper and lower culmination. 6. Combined altitudes of Polaris and stars of nearly equal altitude, *on*, the meridian. 7. By transits across prime vertical. 8. By equal altitudes of sun or star.
LONGITUDE.	Transit of moon, and moon culminating stars. Lunar distances. Occultations.

It is necessary in the first place to understand clearly the following definitions :—

Axis of the Earth is the diameter round which the earth performs its diurnal revolution.

Equator, the great circle on the earth's surface equidistant from the poles, and therefore perpendicular to the axis.

The Terrestrial Meridian of a station on the earth is a great circle passing through both poles and the station.

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The Terrestrial Latitude is the angular distance of a station from the equator measured on its terrestrial meridian,—north or south.

The Terrestrial Longitude is an arc of the equator, intercepted between the meridian of the station and any other fixed meridian, such as Greenwich, Washington, &c., &c., and is reckoned east and west from such meridian up to 180°.

The "Sensible" Horizon is a plane tangential to the *earth's surface at the observer's station.*

The definitions given below, as referring to a star, are equally applicable to the sun or planets.

The Celestial Sphere is an imaginary spherical surface of infinite radius with the earth as a centre.

The Zenith and Nadir are the points of the sphere of the heavens vertically over and under the observer's station.

The Elevated and Depressed Poles, the points indicated on the celestial sphere by the production of the earth's axis, or the vanishing point of a system of lines parallel to it.

The Equinoctial, a great circle of the celestial sphere indicated by the production of the plane of the equator, or the vanishing line of a system of planes parallel to the equator.

The Celestial or "Rational" Horizon, a great circle of the celestial sphere whose poles are zenith and nadir, or the vanishing line of a system of planes parallel to the observer's sensible horizon.

The Celestial Meridian, a great circle passing through the zenith and the pole.

The Prime Vertical is a great circle passing through the east and west points and perpendicular to the meridian.

The Ecliptic, a great circle traced on the celestial sphere by the apparent path of the sun. This circle is inclined at an angle of 23° 28′ to the plane of the equinoctial, and cuts it in two points called the vernal and autumnal equinoxes.

The Vernal Equinox is used as a zero point from which to measure arcs of the equinoctial. At the time this zero point was fixed the sun was in the constellation Aries at the vernal equinox, which consequently got the name of 1st Point of Aries.

Right Ascension of a heavenly body is the arc of the equinoctial intercepted between the vernal equinox and the great circle passing through the pole and the body; generally reckoned in time. Or, in other words, is the angle at the pole contained between the meridian passing through the vernal equinox and the meridian passing through the star, or point observed.

N.B.—When a star is *on the meridian* its RA and the sidereal time of the place are evidently the same—see definition of sidereal time.

Declination is the arc of a great circle, passing through the pole and heavenly body intercepted between the body and the equinoctial. The complement of this arc is the *polar distance.*

Altitude is the arc of a great circle passing through the zenith and the body, intercepted between the body and the *rational* horizon.

Azimuth is the arc of the horizon intercepted between the meridian

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of the observer and a great circle passing through the zenith and the heavenly body.

Latitude of a star is the arc of a great circle passing through the poles of the ecliptic and the star, intercepted between the star and the ecliptic.

Longitude of a star is the arc of the ecliptic intercepted between the vernal equinox and a great circle passing through the pole of the ecliptic and the star.

Solstices, the points on the ecliptic where the sun attains his greatest northern and southern declination.

An Apparent Solar Day is the interval between the two successive transits of the true sun's centre over the same meridian.

As the apparent solar day is a variable quantity (owing to the eccentricity of the earth's orbit, and to the obliquity of the ecliptic) the average of all the apparent solar days in the year is taken as the mean solar day, and a mean sun is supposed to move in the plane of the equator with the average velocity of the true sun, so that a *mean solar day* may be defined as the interval between the two successive transits of the mean sun's centre over the same meridian. This is the day used in civil reckoning.

Apparent Time at any moment is the angle at the pole (in time) contained between the meridian of the place of the observer and the meridian passing through the sun's centre.

Mean Time at any moment is the angle at the pole (in time) contained between the meridian of the place of the observer and the meridian passing through the mean sun's centre.

Equation of Time at any moment is the angle at the pole contained between the meridians passing through the centres of the mean and true suns; or, in other words, the amount to be added to, or subtracted from, apparent time to obtain from mean time.

A Sidereal Day is the interval between the two successive transits of a star across the same meridian.

Sidereal Time at any moment is the angle at the pole contained between the meridian of the place and the meridian passing through the vernal equinox or 1st Point of Aries.

Sidereal time is reckoned from 0 to 24 hours, whereas mean time is only reckoned up to 12 hours.

Astronomical Time is mean time reckoned from the preceding noon up to 24 hours.

Refraction.—Every ray of light passing into a denser medium is bent downwards, consequently the position of an object in the heavens appears to the observer higher than it really is. This amount of displacement is called *refraction*.

The displacement due to the whole atmosphere is called *celestial* refraction, that due to the difference of atmosphere between two stations on the earth's surface is called *terrestrial*.

Parallax is the angle subtended by the earth's radius at the centre of any heavenly body, and is the correction to be applied to one fixed point, viz., the earth's centre.

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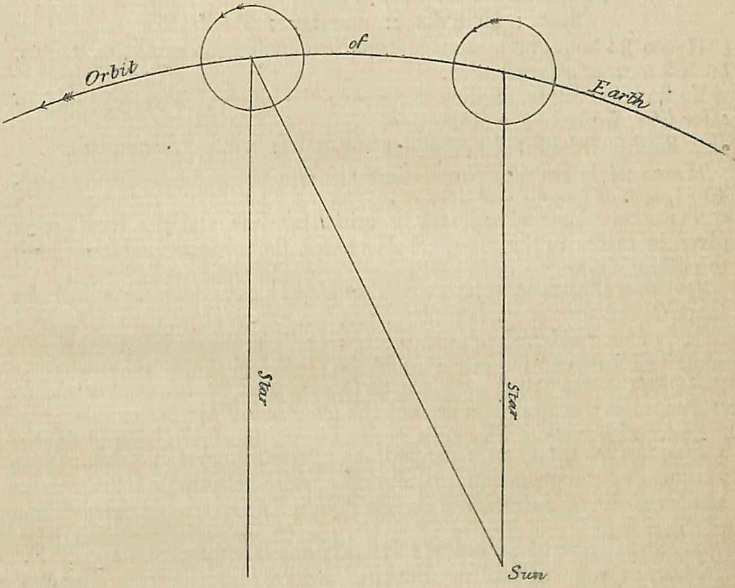
Diagram shewing that a Solar day is longer than a sidereal day :—



We have seen that the interval between the two successive transits of a star over the same meridian is called a *sidereal day*.

The interval between the two successive transits of the sun's centre over the same meridian is called a *solar day*.

The sun completes an apparent revolution about the earth in one year, or 365 days, 5 hours, 48 minutes, and 47·57 seconds, so that the sun's mean daily motion is found by the proportion,—

$$\text{One year} : \text{one day} :: 360^\circ : \text{daily motion}$$

$$365{\cdot}242216 : 1 :: 360^\circ : 59' \, 8{\cdot}33''$$

This motion is not really uniform, but is greatest when the sun is nearest the earth.

Hence the apparent solar days are unequal, and to avoid the inconvenience which would result from this fact, astronomers have recourse to a *mean* solar day.

The length of the mean solar day is different from that of the sidereal, because when the meridian in its diurnal motion, returns to the mean sun, the latter is $59' \, 8''{\cdot}33$ advanced eastward in right ascension.

An arc of the equator, equal to $360^\circ \, 59' \, 8''{\cdot}33$, passes the meridian in a mean solar day, while only 360° pass in a sidereal day—see figure.

To find the excess of the solar above the sidereal day, *expressed in sidereal time*, we have the proportion,

360° : 59′ 8″·33 :: one day : 3′ 56″·555

Hence 24 hours of mean solar time are equivalent to 24 hrs. 3 mins. 56·555 secs. of sidereal time.

To find the excess of the solar above the sidereal day, *expressed in solar time*, we have the proportion,

360° 59′ 8″·33 : 59′ 8″·33 :: one day : 3 mins. 55·909 secs.

Hence 24 hours of sidereal time are equivalent to 23 hrs. 56 mins. 40·91 secs. of mean solar time.

The equivalents of sidereal time in solar time, and *vice versâ* being given in tables in the *Nautical Almanack*, the conversion of one into the other is a very simple matter.

But to understand this conversion clearly, two definitions must be remembered—

1st. The "*sidereal time at mean noon*" is the angle at the pole contained between the meridian of the place and the meridian passing through the 1st Point of Aries, at the moment of mean noon, *i.e.*, at the moment the mean sun crosses the meridian of the place.

2nd. The "*mean time of sidereal noon*" or of the transit of the 1st Point of Aries, is the angle at the pole contained between the meridian of the place and the meridian passing through the centre of the mean sun at the moment the 1st Point of Aries crosses the meridian of the place.

Both these are given in the *Nautical Almanack*, pages II. and XIX.

Now, suppose we want to find the "sidereal time at mean noon" at Chatham on the 14th March, 1870 :—From N A we find, sidereal time at mean noon at Greenwich is 23 hrs. 27 mins. 36·39 secs., and we must subtract the proportionate part of the difference between a sidereal day and a mean solar day, due to our difference of longitude E, in sidereal time. Now our longitude is 128·8 secs. E, and the proportionate part due to this will be—

$$\frac{3' \, 56''\cdot555}{\text{24 hours in secs.}} \times 128\cdot8 \text{ secs.} = \cdot35263 \text{ of a second (to be subtracted).}$$

Again, being given the "mean time of sidereal noon," or transit of 1st Point of Aries, on the 14th March, 1870, at Greenwich, to find it for sidereal noon at Chatham on same date.

Page XIX., *Nautical Almanack*, we find it Greenwich to be 0 hrs. 32 min. 18·31 secs., and therefore to find it at Chatham we must add—

$$\frac{3' \, 55''\cdot909}{\text{24 hours in secs.}} \times 128\cdot8 = \cdot35168 \text{ to obtain it for Chatham.}$$

Now to convert sidereal into mean solar time :—

To the mean time of the preceding sidereal noon (page XIX., *Nautical Almanack*,) add the mean interval corresponding to the given sidereal time, taken from the table of equivalents.

Example.—Convert 11 hrs. 10 min. 11 secs. sidereal time, 14th March, 1870, into mean time. Longitude E, 2 min. 8·8 secs.

	hrs.	mins.	secs.
Mean time of preceding sidereal noon, Greenwich, page 19	0	36	14·21
Correction for longitude, E ...			·35
Mean time at preceding sidereal noon at Chatham...	0	36	14·56
Mean interval	11	8	21·206
Mean time required	11	44	35·766

From table of equivalents :—

	hrs.	mins.	secs.
11 hrs. =	10	58	11·8748
10 mins. =		9	58·3617
11 secs. =			10·9700
	11	8	21·2065 Mean interval.

To convert mean time into sidereal time :—

Rule.—To the sidereal time at the preceding mean noon (page II.) add the sidereal interval corresponding to the given mean time, taken from the tables of equivalents (*Nautical Almanack*).

Example 1.—Convert 9 hrs. 30 mins. 10 secs. mean time, *p.m.*, 14th March, 1870, (longitude E, 2 min. 8·8 secs.) into sidereal time.

	hrs.	mins.	secs.
Sidereal time at the preceding mean noon at Greenwich, 14th March, 1870)	23	27	36·39
Correction for longitude			·35
Sidereal time at mean noon at Chatham	23	27	36·04
Sidereal interval	9	31	43·6639
Sidereal time required.........	8	59	19·7039

From table of equivalents :—

	hrs.	mins.	secs.
9 hrs. =	9	1	28·7083
30 mins. =		30	4·9282
10 secs. =			10·0274
	9	31	43·6639 Sidereal interval

Example 2.—Convert 9 hrs. 30 mins. 10 secs. mean time, *a.m.*, 14th March, 1870, (longitude E, 2 mins. 8·8 secs.) into sidereal time.

Here 9 hrs. 30 min. 10 secs., 14th March, must be considered as 21 hrs. 30 mins. 10 secs., 13th March, and we shall have—

	hrs.	mins.	secs.
Sidereal time at M.N., Greewich, 13th March	23	23	39·83
Correction for longitude ...			·35
Sidereal time at M.N. at Chatham	23	23	39·48
Sidereal interval	21	33	41·9415
Sidereal time required.........	20	57	21·4215

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From table of equivalents :—

	hrs.	mins.	secs.	
21 hrs. =	21	3	26·9859	
30 mins. =		30	4 9282	
10 secs. =			10·0274	
	21	33	41·9415	Sidereal interval

TRANSITS.

The simplest way of finding the time is with an instrument called a "transit instrument," which is fixed in the plane of the meridian, having only a motion in altitude.

If the time be noted by a clock adjusted to sidereal time, when a star crosses the centre of the telescope, this time should be equal to the known R A of the star (*Nautical Almanack*, page 325—387—see definitions), the telescope being in the meridian.

To ensure the instrument moving exactly in the plane of the meridian, we must have the following conditions satisfied.

1st. The axis on which the telescope turns must be horizontal. (Level error).

2nd. The centre wire must coincide with the optical axis of the telescope. (Collimation error).

3rd. The vertical circle described by the centre wire must be a meridian, *i.e.*, must pass through the pole and the zenith. (Azimuth error).

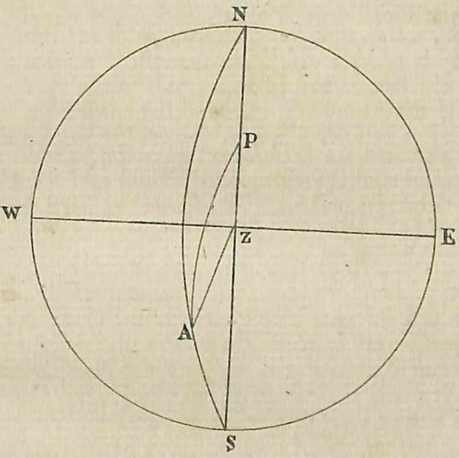
While all of these conditions are evidently necessary and essential, they are all only approximately possible by direct means, but if we *know* the errors of level, collimation, and azimuth, we can correct our observations so as to render them of equal value with observations taken absolutely on the meridian.

LEVEL ERROR.

To find the level error, and having found it, to compute the correction to be applied to the time of transit for this inclination of the axis :—

The spirit level is rested on the pivots of the axis to determine its inclination, but as the legs of the level may not be exactly the same length, it is necessary, after reading the scale at each extremity of the air bubble, to reverse the level end for end and read the scale again as before; then if the number of readings be four, (two east and two west), $\frac{1}{4}$ the difference of the sums of the east and of the west readings will be the amount of deviation from the horizontal in terms of the divisions of the level. In our instrument the value of one division on the level is 15″, or one second of time. Now, having determined the inclination of the axis, to compute the correction to be applied on this account to the observed time of transit.

Let P be the pole
Z the zenith.



Suppose that the transit instrument is in the meridian at N and S, but that the axis is inclined to the horizon by a small angle. Then the telescope, instead of describing the meridian N Z S will describe an oblique circle N A S, and the star A when it passes across the telescope will be distant from the meridian by the small hour angle A P Z, or A P S.

Now in △ A P S, we have,

sin. P A : sin. S :: sin. S A : sin. P,

or cos. dec. : sin. S :: cos. zenith dist. : sin. P,

or P and S being small angles, we shall have

$$P = \frac{S \text{ cos. zenith distance}}{\text{cos. dec.}} \quad (1)$$

where S is the inclination as before determined.

The signs of the corrections as applied to the observed times of transit, are as follows:—When the west end pivot is too high, the correction is to be added, for all stars above the pole, and subtracted when too low. For a transit below the pole the reverse would be the case.

COLLIMATION.

The optical axis of a lens is the line which joins the centres of the spherical surfaces by which the lens is bounded.

The principle line of sight, or *line of collimation* is determined by the direction of the ray of light which passes through the centre of the object glass and touches the middle vertical wire midway between the horizontal wires.

In the rotation of the telescope about its axis, the line of collimation should describe a plane perpendicular to this axis.

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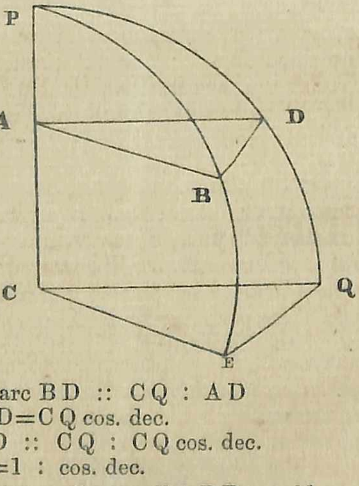
To determine whether this be the case or not, direct the telescope to some distant small well-defined object, and bisect it with the middle vertical wire (or set up a finely graduated scale and read the number which is cut by the centre wire); then carefully reverse the telescope and replace it with its axis reversed. Read the scale (or distant object) again, and if the same graduation be read the collimation is correct; if not, move the wires half the distance between the two readings, by the small screws which hold the diaphragm, and read the scale. Reverse the telescope again and read the scale, and it should read the same as when *last* read; if not, correct half as before, and reverse and correct until in both positions of the instrument the reading is the same. The adjustment will be then complete.

By comparing the observations of transits of different stars it will be seen that the time occupied by a star in traversing the intervals between the several wires is different on different points of the meridian, being least at the equator and increasing with the distance from that circle.

The time occupied by a star on the equinoctial in passing between any two wires is called the *equatorial interval* for the wires, and when this interval is known their interval for any parallel of declination may be computed.

Thus in figure, let P be the pole of the heavens, E Q an arc of the equinoctial, and B D an arc of any parallel of declination, P B E and P D Q two meridians but slightly inclined to each other; a star at B moves over the arc B D in the same time that one at E moves over E Q. But we have (*Euclid*)



$$\text{arc } EQ : \text{arc } BD :: CQ : AD$$
$$\text{but } AD = CQ \cos. \text{dec.}$$
$$\therefore EQ : BD :: CQ : CQ \cos. \text{dec.}$$
$$= 1 : \cos. \text{dec.}$$

Now the time in which a star on the parallel B D would move over a constant space E Q, must be to the time in which an equatorial star moves over the same, inversely as their rates of motion, or as—

$$EQ : BD :: 1 : \cos. \text{dec.} :: \sec. \text{dec.} : 1$$

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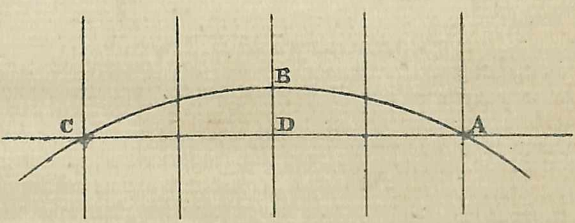
If then x represent the equatorial interval of the wires, x sec. d will be the interval for star whose dec. is d. } EQ : BD (inversely) as their rates of motion.

The equatorial interval may therefore be computed from observations made upon any star whose declination is known, by multiplying the observed interval by the cos. of the star's declination.

More accurately, however, this can be done from observations of a star near the pole—the pole star for instance, but, in this case, a slight modification of the preceding rule is necessary, inasmuch as the pole star does not pass perpendicularly from wire to wire but describes an arc of the small circle A B C.



Now A D is the sine of the arc A B; in order, therefore, to obtain the equatorial intervals from the transit of the pole star, we must multiply the *sine of the observed interval* by the cos. of the declination, and we shall obtain the *sine* of the equatorial interval.

For example, suppose the times over the different wires to have been noted, to be—

	hrs.	mins.	secs.
	0	39	48·0
		48	3·0
		56	19·0
	1	4	32·0
		12	44·0
		20	57·0
		29	16·0
mean	1	4	31·29

The observed intervals from the *mean wire* (which differs slightly from the centre wire) will be—

mins.	secs.
−24	43·29
−16	28·29
− 8	12·29
+ 0	0·71
+ 8	12·71
+16	25·71
+24	44·71

And the equatorial intervals thence deduced will be—

secs.
−38·559
−25·719
−12·820
+ 0·018
+12·830
+25·652
+38·595

being computed as follows :—

24 mins. 43·29 secs. = 6° 10′ 49″·25 sine = 9·0320497
cos. dec. 88° 30′ 27″ = 8·4157426

38·559 secs. = 9′ 38″·39 sine = 7·4477923

and so on for all.

It is customary to reduce all observations to the *mean wire* and not to the middle one.

Now, supposing we miss a wire from any accident, we can find the time of the star crossing it by adding or subtracting the intervals of that particular wire from the wires immediately on its right and left, to the times of transit of the star over the wires immediately on its right and left, if the star be an equatorial star, or, if the star be not on the equinoctial, by multiplying this interval by the secant of the star's declination. Suppose that we miss the 2nd wire, then the time transit over the mean wire is $\frac{t_1+x+t_3+t_4+t_5}{5}$, where t_1, t_3, t_4, and t_5 are the times of transit over the 1st, 3rd, 4th and 5th wires, x being the time over the missing wire, then it is clear that the declination of the star being d, we have $x = t_1 + e$ sec. d. where e is the equatorial interval between the wires, then the time over the mean wire is $\frac{t_1+(t_1+e \text{ sec. } d)+t_3+t_4+t_5}{5}$. (Practically, it will be usually preferable to take another transit.) If the star be near the pole, we must multiply the *sine* of the equatorial interval by the secant of the star's declination, and we shall obtain the *sine* of the reduction to the mean wire.

For instance, suppose 13·816 secs. to be the equatorial interval for any wire, in time;

then 13·816 sec. = 3°27′·15″
log. sin. 3·27·15 = 7·0020488
log. sec. Polaris = 11·5851796
log. sin. 2°12·544 = 8·5872280
= 8m. 51·7 seconds.

In correcting imperfect transits of the sun and planets, the value of the intervals found, as for a star of the same declination, must be increased by a small quantity. For if a fixed star and the sun's first limb were together at the first wire, the sun would be behind the star when it passed the second wire, on account of the sun's apparent motion among the stars. For the sun or a planet, therefore, the interval found for a star must be multiplied by the factor

$$\frac{3600+\mathrm{I}}{3600}$$

where I represents the hourly increase of right ascension in seconds of time taken from the *Nautical Almanack*. For an imperfect transit of the moon the interval found for the sun or a planet must be multiplied by the factor

$$\frac{\text{Sine of moon's geocentric Z. D.}}{\text{Sine of moon's apparent Z. D.}}$$

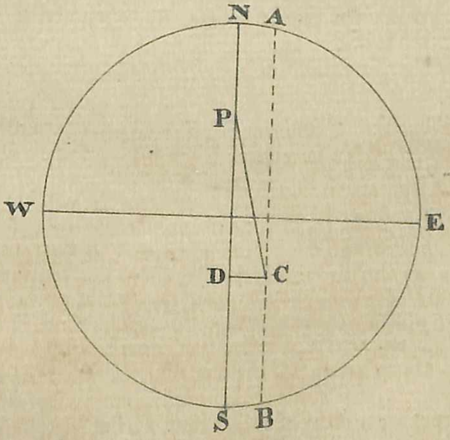
or the entire factor for the moon will be

$$\frac{3600\times\mathrm{I}}{3600}+\frac{\text{Sine of moon's geocentric Z. D.}}{\text{Sine of moon's apparent Z. D.}}\times\text{ secant}$$

of moon's geocentric declination, where I is the hourly increase of the moon's right ascension in seconds of time.

These preliminary explanations are necessary to explain the method of finding any little collimation error that may exist after it has been as far as possible eliminated by the method explained before.

Now, suppose we take the transit of, say, Polaris over two wires and then reverse the instrument and note the times of transit again over the same two in reverse order, and then reduce each of the observations to the mean wire, the mean of the observations in the first position of the telescope ought to be the same as the mean of the two in the reversed position. If the two results differ, it must be owing to an error of collimation.



Suppose the telescope does not move exactly in the meridian, but in a small circle parallel to it A B, and a certain number of seconds east of it. Let P be the pole and C the place of the star. Draw C D perpendicular to N S, the meridian. Then when the star crosses the

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telescope its angular distance from the meridian will be C P D = P. Now,

in △ C P D we have by Napier's rules

$$\sin. CD = \sin. PC \sin. CPS$$

$$\sin. CD = CD \times \sin. 1''$$

$$\sin. P = P \times \sin. 1''$$

$$\therefore CD \times \sin. 1'' = \sin. PC \times P \sin. 1''$$

$$CD = \cos. d \times P \qquad (2).$$

The following example will explain this method.

The transit of Polaris was observed over two wires, then the instrument was reversed and the transit over the same two in this reversed order, observed, as follows:—

$$\text{Corrected for level error.} \left\{ \begin{array}{l} \text{Wire I. 0h. 44m. 33·5s.} \\ \text{,, II. 0h. 52m. 46·0s.} \end{array} \right.$$

Instrument reversed,

$$\text{Corrected for level error.} \left\{ \begin{array}{l} \text{Wire II. 1h. 8m. 56s.} \\ \text{,, I. 1h. 17m. 8·5s.} \end{array} \right.$$

Now, if to each of the above times we add the time the star would take to pass from each wire observed to the mean wire, it is clear that we get the values of the transit of the star over the mean wire in each position of the instrument. The intervals in time to pass from each wire to the mean wire are obtained from the known equatorial intervals by multiplying the sin. of these intervals by the sec. of the declination which gives us the sin. of the intervals, and hence the intervals. In this manner the intervals for the particular star under observations are as follows, viz.:—

16m. 23·59s.
8 12·28

and adding these to the times observed, we get the times of passage over mean wire

$$\left. \begin{array}{lll} \text{1h.} & \text{0m.} & 57·09 \\ 1 & 0 & 58·28 \end{array} \right\} 57·68\text{s.} \left. \right\} \text{Mean time of passage over mean wire in 1st position.}$$

difference 13·375

$$\left. \begin{array}{lll} 1 & 0 & 43·72 \\ 1 & 0 & 44·91 \end{array} \right\} 44·31\text{s.} \left. \right\} \text{Mean time of passage over mean wire in 2nd position.}$$

The difference between the mean of the first two and the last two being 13·37 secs.; half of which (6·685 secs.) represents the angle P = CPD, which, multiplied by the cos. of the star's declination, gives the constant of collimation expressed in seconds of time to be 0·181 secs., which is *minus* in the first position of the instrument, and *plus* in the second position.

Now, having found the constant of collimation of the instrument, to determine the correction to the time of transit for this error.

By the preceding figure, equation 2, we have only to multiply the constant of collimation by the secant of declination.

Example.—The transit of Castor was observed at 7 hrs. 24 min.

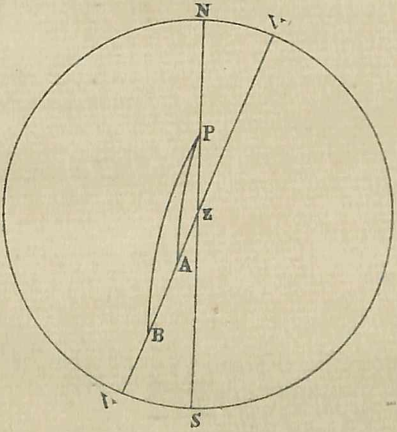
6·52 secs., the constant of collimation being 0·062 secs. minus. Required the correction in time

$$\log \cdot 062 = \bar{2} \cdot 722$$
$$\text{log secant declination} = 0 \cdot 073$$
$$- 0 \cdot 07 \text{ secs.} = \bar{2} \cdot 865$$

∴ corrected time of transit is 7 hrs. 24 min. 6·45 secs.

AZIMUTH.

To determine the deviation of the transit instrument from the meridian, if we take the difference between the observed passages (corrected for level and collimation) of two stars (whose difference of declination is considerable) over the meridian, and also the difference of their computed right ascensions, and if we find these differences to be exactly equal, the instrument will be exactly in the meridian: but if they are not equal this difference shows a deviation from the meridian.



Let N Z S be the true meridian.
V Z V' ,, vertical circle traced by the telescope.
P ,, pole.
Z ,, zenith.
A & B ,, two stars observed.
ϕ = latitude of place.
δ & δ' = declination of the two stars.
Δ = difference of observed times — the difference of R A *s.* = A P B.
a = azimuthal deviation of the instrument called azimuth constant.

Then in Δ Z P A we have

sin. P A : sin. Z A :: sin. P Z A (= sin. A Z S) : sin. Z P A

or cos. δ : sin. Z :: a : Z P A, a and Z P A being small angles.

$$\therefore \text{Z P A} = \frac{a \sin. \text{Z}}{\cos. \delta.} \quad . \quad . \quad . \quad . \quad (a)$$

$$= \frac{a \sin. (\phi - \delta)}{\cos. \delta.}$$

$$= \frac{a\,(\sin. \phi \cos. \delta - \cos. \phi \sin. \delta)}{\cos. \delta}$$

$$= a\,(\sin. \phi - \cos. \phi \tan. \delta)$$

in the same manner we find

$$\text{Z P B} = a\,(\sin. \phi - \cos. \tan. \delta')$$

$$\therefore \text{A P B} = a \cos. \phi\, (\tan. \delta - \tan. \delta')$$

$$\text{or } a = \frac{\text{A P B}}{\cos. \phi\, (\tan. \delta - \tan. \delta')}$$

$$\text{but } \tan. \delta - \tan. \delta = \frac{\sin. (\delta - \delta')}{\cos. \delta \cos. \delta'} \therefore a = \frac{\Delta \cos. \delta \times \cos. \delta'}{\cos \phi \sin. (\delta - \delta')} \quad (3)$$

The sign of Δ is determined from the equation

$\Delta = (\text{T}^n - \text{T}) - (\text{R}^n - \text{R})$ when T^n and R^n are the observed times and the right ascensions of the northernmost star.

Example.—The transit θ Ceti (dec. S. 8° 57′), (corrected for level and collimation) was observed at 1 hr. 16 min. 0·95 secs., and that of Polaris (de[illegible] 88° 30′) at 1 hr. 5 min. 17·63 secs., the difference of the tabulated right ascensions being 10 min. 40·39 secs.; the difference between the observed passages being 10 min. 43·32 secs. To find the azimuth constant. From the differences given $\Delta = -2{\cdot}93$ secs.

$$\begin{array}{lr} & \text{and } 2{\cdot}93 \text{ secs.} = 43{\cdot}95'' \log. = 1{\cdot}6430 \\ & \cos. \delta = 8{\cdot}4158 \\ & \cos. \delta' = 9{\cdot}9947 \\ & \sec. \phi = 0{\cdot}2056 \\ \left.\begin{array}{r} 88^\circ\ 30 \\ 8^\circ\ 57 \\ \hline 97^\circ\ 27 \end{array}\right\} & \begin{array}{r} \text{cosec. } 97^\circ\ 27' = 0{\cdot}0037 \\ \hline a = -\ 1''\ 83 = 0{\cdot}2628 \end{array} \end{array}$$

Any two stars differing considerably in declination will do for this observation, but it is better to take Polaris as one of them, or some close circumpolar star.

The correction to the time of transit for the error of azimuth is according to equation $(a) = \text{Z P A} = \dfrac{a \sin. \text{Z}}{\cos. \delta.}$ (4)

That is the numerical correction in seconds of time to each transit = azimuthal constant (in time) × sin. zenith distance × sec. star's declination.

RULE.

If the eastern pivot is to the south of east, the correction to be added to the observed times of transit for all stars between the zenith and pole, and subtracted for all stars below the pole or south of zenith. If the eastern pivot deviates to the north of east, the corrections will be applied with signs contrary to the above.

Example.—The transit of Castor, dec. 32° 12′ 32″ N, was observed at 7 hrs. 24 min. 6·52 secs., the azimuth error being − 8·32″. Required the corrected time of transit for azimuth error.

Zenith distance was 19° 16′ 7″

$$\begin{aligned} \text{sin. } Z &= 9{\cdot}5185 \\ \text{sec. } \delta &= 0{\cdot}0726 \\ 8{\cdot}32'' = 0{\cdot}5546 \text{ secs.} &= 9{\cdot}7440 \\ \hline - 0{\cdot}22 \text{ secs.} &= 9{\cdot}3351 \end{aligned}$$

∴ corrected time of transit is 7 hr. 24 min. 6·30 secs.

We can also find the azimuth constant as follows :—

There are two stars near the N. Pole which culminate nearly at the same time, one above and the other below the pole, viz., 51 Cephei and δ Ursæ Minoris.

The formula of reduction is the same as for two stars of different declinations as shown above, except that instead of (δ') we must put ($180 - d'$), since one star is below the pole.

The observed transits must first be corrected for level and collimation. Then put Δ = difference of the observed times *minus* the difference of R A*s* neglecting the 12 hours.

The error in azimuth will be

$$a = \frac{\Delta \cos \delta \cos \delta'}{\cos \phi \sin (\delta + \delta')} \qquad (5)$$

Example.—Transit of δ Ursæ Minoris was observed at 6 hr. 19 min. 29·4s., and of 51 Cephei at 6 hr. 28 min. 1.58s. Difference of tabular R As being 12 hr. 8 min. 20·99s. Required the error in azimuth.

$$\begin{aligned} \Delta = 10{\cdot}85 \text{ secs.} = 162''{\cdot}75 \text{ log.} &= 2{\cdot}2115 \\ \cos \delta' &= 8{\cdot}6799 \\ \cos \delta &= 8{\cdot}7737 \\ \sec \phi &= 0{\cdot}2056 \ (\phi \text{ being lat. of Greenwich in this case.}) \\ \text{cosec } 6^\circ\, 8'\, 51'' &= 0{\cdot}9703 \\ \hline a = 6''{\cdot}93 + &= 0{\cdot}8410 \end{aligned}$$

Hence the error in azimuth is 6″·93 + and since the observed interval between the stars was too great, it is evident that the telescope pointed to the west of the true meridian on the north horizon or to the east of south.

INEQUALITY OF PIVOTS.

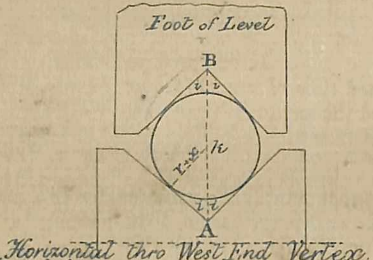
If the axis of the transit was a perfect cylinder, and line the joining points of the Y's in which it rests horizontal, then the axis of the transit would be a horizontal line ; but owing to imperfections in workmanship, or unequal wear, etc., this is seldom the case ; and the inclination which the axis has from these causes necessitates a certain correction which we will proceed to find,—

Let $2i$ be the angle of the Y's of bearings and $2i'$ the angle of the Y's of level.

Let r be the radius of the smaller pivot and $r + x$ that of the larger or perforated end of axis.

Let c be the elevation of the vertex of the angle of eastern Y above that of the western.

Then the elevation of the vertex of the east-end level above the horizontal passing through the west vertex $=c+AB-A'B'$.



Now, $AB=(r+x)(\text{cosec } i+\text{cosec } i')$,
and $A'B'=r(\text{cosec } i+\text{cosec } i')$; and therefore
$c+AB-A'B'=c+x(\text{cosec } i+\text{cosec } i')$.

And if the length of the axis of the transit be taken as the unit of distance, the above quantity represents the angular elevation of the east end of instrument above the west. This angular elevation is found from the readings of the level in its 1st and 2nd positions. Calling these readings R and R′, we have

$$R=c+x(\text{cosec } i+\text{cosec } i) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

Now, reverse the transit in its bearings and we have then—
Height of B above $A'=(r+x)(\text{cosec } i+\text{cosec } i')$, and
do. B' above $A'=\ +x(\text{cosec } i+\text{cosec } i')$
and the difference between B and datum
is $R'=c-x(\text{cosec } i+\text{cosec } i)$ (b) which is the angular elevation on the second supposition.

Substract and add b and (a) we get—

$$x=\frac{R-R'}{2(\text{cosec } i+\text{cosec } i')}=\text{difference between radii pivots.}$$

$$\text{and } c=\frac{R+R'}{2}$$

and the elevation of the east end of axis above west, when perforated end is east, is—

$$c+x\,\text{cosec } i=c+\frac{(R-R')\,\text{cosec } i}{2(\text{cosec } i+\text{cosec } i')}$$
$$=\frac{R+R'}{2}+\frac{(R-R')\,\text{cosec } i}{2(\text{cosec } i+\text{cosec } i)}$$

If we suppose $c=o$ we have then

$$x=\frac{(R-R')\,\text{cosec } i}{2(\text{cosec } i+\text{cosec } i')} \qquad (6)$$

and furthermore if the $i=i'$ we have

$$x=\frac{R-R'}{4}$$

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TRANSIT INSTRUMENT. INEQUALITY OF PIVOTS.

Vertical Circle.	Level Readings. East End.	West End.	Means,	
East	8·99	68·4		
	8·45	72·9		
	9·03	6·71		
	8·34	73·2	+·831	
West	7·98	7·64		
	8·69	6·98		
	8·01	7·64		
	8·65	7·00		+·509
East	8·15	7·46		
	8·22	6·72		
	8·24	7·36		
	8·95	6·66	+·757	
West	8·15	7·43		
	8·90	6·71		
	8·33	7·26		
	8·91	6·68		+·775
East	8·22	7·39		
	9·04	6·59		
	8·51	7·09		
	9·07	6 57	+·900	
West	7·87	7·76		
	8·49	7·12		
	7·82	7·82		
	8·56	7 10		+·367
East	8·47	7·19		
	9·01	6·62		
	8·33	7·32		
	9·03	6·65	+·882	
West	7·91	7·75		
	8·61	7·06		
	7·89	7·75		
	8·53	7·16		+·402
East	8·58	7·10		
	8·00	6·80		
	8·50	6·68		
	88·8	6·84	+·817	
West	8·24	7·46		
	8·38	7·36		
	8·14	7·57		
	8·47	7·29		+·444
		5	4·187	2·497
			·8374	·4994
			·4994	

Difference of average,... ·338

Angle of Y of instrument=97° that of level feet=90°

$$\frac{\cdot338}{2}=\cdot169 \text{ log.}=\bar{1}\cdot2278867$$

$$\text{Cosec}\,\frac{97^\circ}{2}=\text{cosec } 48^\circ\,30'=1\cdot3352$$

$$\text{Cosec}\,\frac{90^\circ}{2}=\text{cosec } 45^\circ=\frac{1\cdot4142}{2\cdot7494} \text{ log.}=\underline{0\cdot4392379}$$

$$2\cdot7886488$$

$$\text{log. cosec. } 45^\circ=\underline{10\cdot1505150}$$

$$\text{log. } \cdot086928 = \bar{2}\cdot9391638$$

Correction to level reading=·0869288

By which we find that for our instrument if A be the illuminated end, the pivot at A is the larger, and when east the difference ·09 must be subtracted from the level reading, and if west added.

COMPARISON OF CLOCK AND CHRONOMETER.

The error of the clock having being ascertained by the comparison of the time of transit of the sun or star corrected for level, collimation and azimuth with the computed time of transit, as shown by the *Nautical Almanack*, if we note the coincidence of any instant of time by each, we can easily deduce the error of the chronometer from the known error of the clock. For, the sidereal time at mean noon being given in the *Nautical Almanack*, if we correct this for longitude we get the sidereal time at mean noon at the place of the observer, and subtracting the clock time of comparison (corrected for the known error) from this, or this from the clock time of comparison corrected, we get the interval in sidereal time before or after mean noon, and turning this interval into mean time by the tables of equivalents (see *Nautical Almanack*, page 506), we get the interval from mean noon in mean time, which compared with the chronometer time of comparison gives the error of the chronometer.

ROYAL ENGINEER OBSERVATORY.

H.	M.	S.	
14	11	49·6	Sidereal time at mean noon at Greenwich, Oct. 24th, 1873.
		·35	Correction for Longitude of R.E. Observatory.
14	11	49·25	Sidereal time at mean noon at Observatory.
13	42	11·83	Clock time (corrected) of comparison with chronometer.
	29	37·42	Difference.
	28	55·2490	Mean equivalents for above difference.
		36·8990	
		·4188	
	29	32·5668	Mean interval from noon by clock.

H.	M.	S.	
11	30	27·4332	Mean time A.M. by clock (subtracting from 12).
11	26	0·	„ „ „ chronometer.
	4	27·4332	Slow.
	(4	26·8	By observations).

Clocks are compared by comparison of coincident beats. If the two chronometers keep the same sort of time, it will generally happen that the beats are not synchronous, but will differ by a certain fraction of a second which will be nearly constant, the ear must estimate this fraction which it can do within $\frac{1}{4}$ of a second or $\frac{1}{2}$ a beat if the chronometer beat $\frac{1}{2}$ seconds, but it requires much practice to estimate the fraction within 0·1 seconds with certainty. If, however, while one clock beats solar the other beats sidereal time, the difference between them may be ascertained to within ·05 seconds of time. Since 1 second sidereal time equal ·99727 mean solar—the sidereal gains ·00273 seconds on the solar in 1 second, and therefore it gains ·5 seconds or one beat in $183^{s\cdot}$ or one beat in 3 minutes. Hence every 3 minutes there will be coincidence of beats. When this is about to occur the observer counts the seconds of one chronometer while he fixes his eye on the other, when he judges that a coincidence of beats has taken place, he notes the corresponding $\frac{1}{2}$ seconds of the instruments. The best way of comparing two chronometers beating $\frac{1}{2}$ seconds mean solar time is to compare each with a $\frac{1}{2}$ second or second clock regulated to sidereal time, in case of mean time and sidereal chronometers.

The accuracy of the comparison may be tested: note corresponding times at two successive coincidences, convert the mean solar time interval into sidereal time and add the converted interval to the sidereal time of the first coincidence, the sidereal time so found, should coincide with the sidereal time of the second coincidence.

ON FINDING THE RATES OF TIMEKEEPERS.

If the errors of a watch or chronometer be found as shewn, that is by comparison of clock and chronometer, at a given place, on different days, sufficiently distant from each other, its daily rate may thence be readily deduced upon the assumption of an uniform motion during the intervening period: thus, if the chronometer be found too fast at the time of observation, and the error increasing, the instrument will obviously be gaining on mean time, but if decreasing, it will be losing on mean time.

Suppose on a given day, a chronometer is found to be 25 seconds fast, and at the end of ten days 20 seconds slow, its daily rate is 4·5 seconds losing. Hence the rate of a chronometer, is the quotient of the total gain or loss of the instrument, during the interval between observations, divided by the number of days, of which the interval consists. Observations for the purpose of determining the rate of the chronometer, in order to prove the regularity of its going, should be made as often as convenient, for which purpose the following is a very easy method.

To find the rate of a chronometer, without reference to its error, by equal altitudes of the same fixed star, observed at the same place, and on the same side of the meridian, on different nights.

Rule.—Multiply 3m. 55s·909 (the daily acceleration of the fixed stars on mean time) by the number of days between the observations, and subtract the product from the time of the first observation; divide the difference between the remainder and the time of the last observation, by the number of days, and the quotient is the rate, *gaining*, if the remainder be *less* than the time of the last observations, but *losing* if *more*.

Example.—Suppose on February 4, at 10h. 15m. 26s. and on February 12, at 9h. 43m. 38s. a star on the same side of the meridian had equal altitudes; required the rate of the chronometer.

	H. M. S.	
	10 15 26	First observation.
3m. 55s·909 × 8 days =	31 27·272	Reduction.
	9 43 58·728	Remainder.
	9 43 38	Second observation.
	8 | 20·728	
	9·591$^{s.}$	Losing daily.

N.B.—If the readings of the thermometer and barometer are very different at the times of observation, of course the result will not be quite correct.

THE SEXTANT.

The sextant, as its name implies, is the sixth part of a circle, and contains 60°, but on account of the double reflection, the arc is divided into 120°.

The principal adjustments are as follows:—

1st.—*To make the Index Glass Perpendicular to the Plane of the Instrument.*

Move the index forward to about the middle of the limb, that is, till it reads about 60°; then holding the instrument with the divided limb from the observer, and the index glass to the eye, look obliquely down the glass, so as to see the circular arc by direct vision and by reflection in the glass at the same time; and if they appear as one continued arc of a circle, the index glass is in adjustment, that is, is perpendicular to the plane of the instrument. If the reflected limb appear lower, the glass leans backwards; if higher, it inclines forward; turn the screws in the plane behind the index glass, until the appear in one, and the glass is adjusted.

As the glass is, in the first instance, set right by the maker, and firmly fixed in its place, its position is not likely to alter, except by violence.

2nd.—*To set the Horizon Glass Perpendicular to the Plane of the Instrument.*

Bring the two zeros together, making the coincidence perfect by means of the tangent screw, then fix in the telescope, and by the screw at the back of the instrument, bring the telescope into such a position that its field is bisected by the line which separates the silvered and transparent parts of the horizon glass; now hold the sextant vertically, and direct the sight to the sea horizon, and if the reflected and true horizons appear in one unbroken line, the glass is adjusted, but if they do not, make them coincide by means of the tangent or other screws, which are sometimes placed at the back of the horizon glass; the instrument thus adjusted.

3rd.—*To find the Index Error.*

When the zero on the index is set to zero on the limb, the horizon and index glasses should be parallel; and if the telescope be directed to a star, the two images should exactly coincide. If the two images do not coincide, the deviation constitutes what is called *index error.* The amount of index error may be found in the following manner:—Since the sun subtends an angle of about 30′ at our eye, if we set the index to about 30′ on the arc and make contact, read the arc, and then set the index to about 30′ or so, on the arc of excess. Again make contact, read the arc; then take the difference of the two readings and divide by the number of observations, we shall get the index error, +, if on the graduated arc, the reading be the larger, and − if larger on the arc of excess, this is proved as follows:—

Let x = the index error, R the reading on the arc, R′ the reading off the arc, and let D diameter of sun. Then if $R > D$ we have

$$R = D + x$$
$$R' = D - x$$
$$\frac{R - R'}{2} = x$$

If the readings of the sun's or moon's upper and lower limbs are both on the same side of zero, then add the readings together and divide by two to obtain the index error.

Example.—Suppose the

$$\begin{array}{lrrl} \text{Reading on the arc} & 30' & 46'' & \\ \text{Reading off the arc} & 30 & 12 & \\ \hline & 2\,| & 34 & \text{Difference.} \\ & + & 17 & \text{Index error.} \end{array}$$

again suppose the

$$\begin{array}{lrr} \text{Reading off the arc} & 28' & 22'' \\ \text{,, \quad ,, \quad ,,} & 38 & 36 \\ \hline 2\,| & 66 & 58 \\ - & 33 & 29 \end{array}$$

N.B.—+ correction means − index error and *vice versâ.*

It is necessary always to take the index error *before* and *after* each observation, the mean being taken as the true error; and if at night and the temperature has changed much since the day-determination of the error, it is better to combine the observations to get rid of the

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effect of this cause of error altogether, or it may be approximately found by directing the telescope to a star, the index being set to zero; of course, if there is any error the stars will appear separate, and when brought into coincidence, with the slow motion screw, the reading on the arc will give the error at once—approximately. If the moon is *full*, or nearly so, of course it is as convenient as the sun for ascertaining the index error.

4th.—*To set the Axis of the Telescope Parallel to the Plane of the Sextant.*

Screw in the telescope, and turn the tube containing the eye glass, until the two wires are parallel to the plane of the instrument, then select two distant objects from 130° to 120° apart, as two stars of the 1st magnitude, or the sun and moon: make the contact of these objects as perfect as possible at the *lowest* wire, or that nearest the plane of the sextant; then move the instrument, so as to make the objects appear at the *upper* wire, and if the limbs appear to separate, the object end of the telescope is too low, but if they partly cover each other, it is too high; to rectify the first error, tighten the screw of the double collar of the telescope support, that is *nearest* the sextant, and slacken the more distant one: in the latter case this operation is reversed; should the moon be one of the objects, this adjustment must be made as rapidly as possible, otherwise allowances must be made for the apparent motion in the interval.

If, at the time of taking the observations, the sun's altitude be less than 20°, the *horizontal* instead of the vertical diameter should be taken, on account of the increased refraction near the horizon.

ARTIFICIAL HORIZON.

When it is required to measure altitudes on shore an artificial horizon must be used; it consists of a small trough or basin, containing quick silver, which is protected from the wind by a roof, the opposite sides of which are of plate glass, the sides of each glass being perfectly plane and parallel; or the mercury may be kept still by a piece of plate glass, similarly ground, being laid on the surface, the image reflected from this surface appears as much below the true horizontal line, as the object itself appears above it, consequently the angular distance between the object and the reflected image, corrected for index error, is double the true angle; half this angle therefore, corrected for refraction, parallax, and semidiameter, will be the true altitude of the sun's centre, supposing the observation to be made on the sun. When the inverting telescope is used, the upper limb will appear as the lower, and the lower as the upper. The artificial horizon best calculated to secure clear mercury and a bright image, is that so constructed that the mercury is passed from the bottle into a hole in the upper edge of the trough, which running underneath, rises perfectly clear and filtered from the bottom.

The whole secret of observing well with a sextant, or any hand instrument, is to be able quickly to hold the instrument in the *true plane.*

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The best plan is, after having succeeded in getting the images into the field, to note the position of the arm and wrist, so as to naturally hold the instrument similarly another time. By casting the eye down on to the concave surface of the graduated arc, it can at once be ascertained whether the instrument is being held in the proper plane (if the sun be observed) as the *shadow* of the lower radial bar will just cover the arc when the instrument is in the true plane, and then by a movement up or down of the limb, the reflection of the sun *must* come into the field. It will be found of great use to eliminate as far as possible errors likely to arise from incorrect estimates of refraction and index error by *combining* or balancing observations, *i.e.*, taking pairs on each side of the zenith, north and south, or of the meridian, east and west. Also, where practicable, to let the observations be made symmetrical as regards both the intervals of time and spaces of arc, (but altitudes taken at a greater interval than five minutes should not be grouped together to form a single observation, as the arithmetical mean of them all will not correspond to the arithmetical mean of the time, owing to the fact that the motion of a celestial body in altitude varies in its rate with its altitude). For instance, suppose we are observing the sun when rising, and get the reflected image in the field in the proper position at 70°—Set the index to 70° 10′ and wait until contact is made by the sun altering in altitude.—Note the time when this takes place, and set the index to 70° 20′.—Wait for contact again and note the times, and so on (waiting for contact is only practicable when the celestial body is altering its altitude rapidly; when it is near the meridian the motion is too slow). Of course exactness in setting the arc is necessary, but if the eye is truly opposite the graduation, no difficulty will be found. The angle need not be written down at all, save once, as all the others will be at 10′ intervals, (assuming that we are taking 10′ intervals) and the times will be nearly at equal intervals, so that errors may be checked. Further, it will be found to be much easier to *wait* for contact when the images of the sun, &c., are approaching one another, than when they are separating and overlapping, so that that limb should always be observed which will give this result; evidently, with the same kind of instrument. the *opposite* limbs will be preferable in the morning and afternoon observations, *i.e.*, when the object is rising or falling.

When the contact is made at the lower limb, the image will *separate* soon after contact has been made, if the altitude be *increasing*, but if the altitude be decreasing, they will begin to overlap; of course, in observations made with the upper limb, this will be *vice versâ*.

In observing an altitude of the moon, great care must be taken to observe the altitude of her *roundest* limb; sometimes, (but only seldom) shortly after new moon, distances may be measured from the moon's *dark* or unenlightened limb, when this can be done there is every reason to believe it is the most accurate method of observation.

About full moon, and when the moon appears quite round to the eye, a mistake may readily be made, and distances be observed from the wrong limb of the moon, but as the *Nautical Almanack* in page 12 of

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each month gives the Greenwich time of full moon, to the tenth of a minute, it is easy to avoid such an error.

All observations for time should be made when the celestial object is due or nearly E or W. In observing lunar distances it will be found easiest to bring *on* the distance of the moon, and whatever object may be observed with it, by looking through the ring of the telescope support, and bringing the object as near as possible, by the naked eye, then screw in the telescope, and looking through it, make contact perfect by the tangent screw. The telescope should of course be set to the proper focus, which will be the case when on looking at a fixed star it will appear *smallest and brightest.*

In taking lunar distances it is better to direct the telescope to the fainter object, particularly if a star, as it can be more easily kept in view when seen directly, than by reflection; should the bright object be to the left, the sextant must be held face downwards. The enlightened limb of the moon must always be brought in contact with the sun or star, even when the moon's image is made to pass beyond the sun or star, before contact can be made.

In taking distances it is best not to make the contact perfect by the tangent screw, but when the nearest limbs are observed, to make the object overlap each other a little when they are receding, or leave a small space between them when they are approaching and wait till contact is perfect; the reverse must be practised when the farthest limbs are observed.

It is never necessary to count or remember the time when observing. When contact or coincidence is made count 0, 1, 2, 3, and so on with the beats of the clock, chronometer, or watch, then turn the head slowly till you see the face of the time piece, and mark when the second hand arrives at some main division, such as 15, 20, 45, &c., and then subtract from this, the number of beats you have counted; note *then* the minute and the hour and write them down. If the chronometer beat half seconds of course only subtract half the number of beats counted. Never forget to write down the place of observation, the date, the kind of instrument used, the index error, the upper or lower limb (if the sun or moon be observed), the thermometer and barometer readings. Never forget to put up the *dark glasses* or use the coloured eye piece whenever observing the sun, it being almost certain loss of sight to omit this precaution; also, focus the telescope on the *object to be observed.* If you have any doubt as to which limb you are taking, *try* the readings which result from altitudes with the sun in reversed positions, which will at once settle the question: you can always tell whether it is the reflected image, or that seen direct in the mercury, by simply moving the head back or forwards in the direction of the plane of observation, as that seen direct in the mercury will disappear, while the reflected image will remain, as long as you remain in the plane of observation.

It will be found of the greatest practical advantage to observe the *same stars* over and over again, night after night. The logarithms will be partly constant, and the star will become so familiar as to be caught with the greatest ease.

Observations made with the sextant, and artificial horizon are the apparent double altitudes of the upper or lower limb of the body observed if it has a sensible diameter, and from these in order to get the true altitudes of the body as seen from the centre of the earth, the undermentioned corrections, and in the order given must be applied to the observed double altitudes.

1st.—Correction for index error to the double altitude—the apparent altitude is then obtained by dividing the result obtained by 2.

2nd.—Correction for refraction, always a minus quantity.

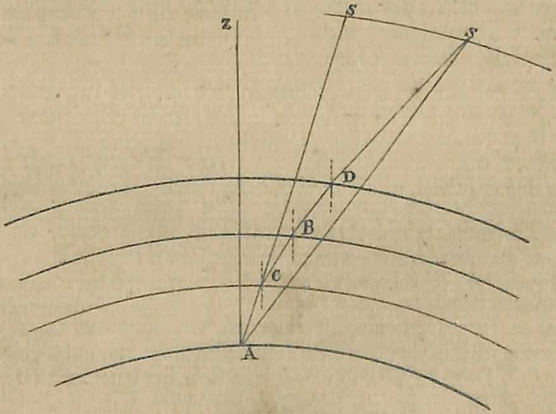
3rd.—Correction for semi-diameter with the sign + or − according as the lower or upper limb has been observed.

4th.—Correction for parallax always a + quantity.

In the case of observations when the sea horizon is used instead of the artificial horizon, and the eye of the observer is situated at a known height above the sea level, the observation taken must be reduced to the level of the sea, this correction, called reduction to the horizon is always a minus quantity.

REFRACTION.

When a ray of light passes through a medium of uniform density it proceeds in a straight line without any deviation, but if it passes obliquely from one medium to another of different density its direction is changed and the deviation from the former course is called the refraction.



Let A be a spectator on the earth's surface and S the position of a heavenly body. Conceive the atmosphere to be divided into successive strata D B C A, etc., concentric with the earth's surface, and suppose these to be so thin that the density of each strata may be considered constant, then as the density of these increase from the outside towards the centre, and if we conceive normals at the points D, B, C, etc., the ray of light which ultimately reaches the observers eye will suffer a deviation in passing through each strata, and in each case the angle it makes with the normal before penetration is greater than that after, the two being connected by the formula sin. $O D S = \mu$ sin. $O' D B$, so that the path described will be a polygon, and if the strata be taken sufficiently close this polygon becomes a curved line, and the direction in which we see the body is in the tangent to the last element of this curve or in the direction $A S'$ and the angle between S and S′ is the refraction and $Z A S = Z A S - S' A S$ or $Z = Z' - r$, or the true zenith dist. = the apparent zenith dist. − the refraction. From an inspection of the figure it is evident that this angle is greater at the horizon and nothing at the zenith, also that there would be no refraction for a spectator situated at the centre of the earth. At the horizon the refraction is about 35′, or greater than the diameter of the sun, so that when we see the sun above the horizon it is nearly just under, thus, the refraction increases the length of our days, as we see the sun before it rises and after it sets. At 45° the refraction is only about 57″. The oval figures of the sun and moon when on the horizon are explained by the rapid increase of refraction when near the horizon, as when they are on the horizon the refraction of the lower limb is about 3″ greater than the upper limb, and hence the vertical diameter is less than the horizontal by that amount—any other diameter is decreased in the ratio of the $\cos^2$ of the angle it makes with vertical.

Tables of refraction have been formed on the following hypothesis—

1st.—That the thickness of the atmosphere is inconsiderable compared to the radius of earth.

2nd.—That the density of the air is uniform.

Hence follows that for small zenith distances, the refraction $\propto$ as the tangent of the zenith distance. For larger Z distances, the refraction thus computed require a correction varying as the cube of the tangent of the zenith distance. In very great Z D still further corrections are required, and below 85″ the amount becomes extremely uncertain.

Bradley's formula is $r = C \tan. (Z - \alpha\, r)$ where $C\, r$ is the mean refraction, C a constant determined from observation, Z the apparent zenith distance, and α a co-efficient, and dertermined by Bradley as 3.

This constant C has been determined from numerous observations of circumpolar stars, γ Draconis for instance, which passes the meridian very near the zenith of Greenwich. Let Z and Z′ be the observed zenith distances $c f (Z')$ and $c f (Z)$ the refraction, then the true zenith distances are

$Z + c f (Z)$ and $Z' + c f (Z')$ and if ϕ be the co-latitude of the place we have

$$2 \phi = Z + Z' + c \left(f(Z) + f(Z') \right)$$

hence if ϕ is known, we have

$$c = \frac{2 \phi - (Z + Z')}{f(Z) + f(Z')}.$$

GSL

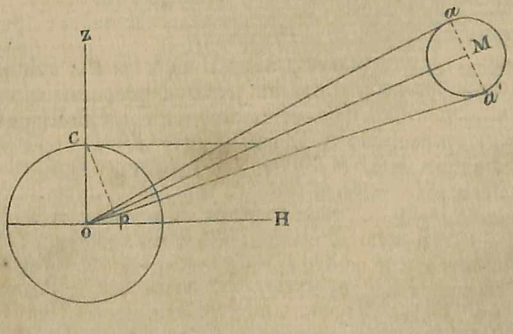
Bessels are the best modern tables of refraction.

If the temperature and pressure differ from 50°, and 29·6 inches the temperature and pressure for which mean tables of refraction are formed, a correction for these two quantities will have to be applied; and it is clear that if the pressure be greater than 29·5, the density of the air must be greater, and hence the refraction must increase; if the temperature be less than 50°, the refraction increases for the same reason, and hence the correction to the mean refraction would be additive—if the pressure be less than 29·6 and temperature greater than 50°, the correction to mean refraction must be subtracted.

SEMI-DIAMETER.

The observed altitudes of the sun moon or planets are usually those of the upper or lower limb, and to obtain from these values the altitudes of the centre, the values of the semi-diameter as seen from the centre of the earth has to be either subtracted or added. For the sun this value is obtained from the *Nautical Almanack* for each day at Greenwich mean noon; owing to the sun's great distance from the earth, compared with the earth's radius, this value is the same whether referred to the centre or circumference; but for the moon, as its mean distance from the earth is only 60 times the earth's radius, it is clear that when it is in the zenith it is nearer to an observer than when on the horizon by a radius of the earth, or $\frac{1}{60}$th its mean distance, and hence the apparent value of the moon's semi-diameter increases from the horizon to the zenith, so that for every altitude above the horizon a correction has to be applied. This correction may be obtained as follows:—

In the figure below, let $O\,M = D$
" " $O'\,M = d$
OO' the radius of earth $= r$
$s =$ the horizontal semi-diameter at time of observation.
(to be found from *Nautical Almanack*).
and the increase of semi-diameter due to elevation above horizon corresponding to distance $d = x$.



Now it has been found from observation that the angular value of moons semi-diameter when at a mean distance from the earth of 60 times the earths radius = 930″, and hence its value when at any distance D is found from proportion.

$$D : 60\,r = 930 : s$$

$$\text{and } \therefore\ D = \frac{55800\,r}{s} \qquad (1)$$

Let fall the perpendicular O′ P on O M and we have

$$OM = OP + PM$$

and O P = O O′ cos. O′ O P = r sin. M O H = r sin. A, and since the ∠ O′ M O is small compared with O′ O M O′ M = M P

$$\text{hence } OM = D = r \sin. A + d \qquad (2)$$

also since the angular value of the semi-diameter is inversely as the distance, we have

$$s : s + x = d : D \qquad (3)$$

$\therefore\ D\,s = d\,s + d\,x$ but from equation (2) $D\,s = r\,s \sin. A + d\,s$ and $\therefore$ from (2) and (3) $r\,s \sin. A + d\,s = d\,s + d\,x$

$$\therefore\ x = \frac{r\,s \sin A}{d} \qquad (4)$$

$$\text{hence } d = \frac{55800\,r}{s} - r \sin. A$$

$$= \frac{55800r - r\,s \sin. A}{s}$$

substitute this value of d in equation (4) and we get

$$x = \frac{r\,s \sin. A}{\frac{55800\,r - r\,s \sin. A}{s}} = \frac{s \sin. A}{55800 - s \sin. A}$$

$$= s^2\,(\cdot 0000\,179 \sin. A \text{ nearly.})$$

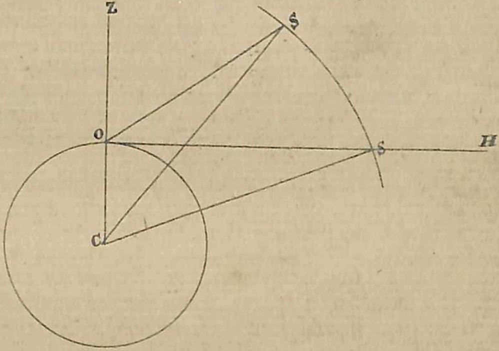
The moon's horizontal semi-diameter is given in page III. *Nautical Almanack* for each day at noon and midnight, and hence knowing its variation for 24 hours, its variation for any time it can be tound by interpolation, and hence the value of x by substituting in above formula. Tables are given in books on astronomy by which this increase is at once found.

PARALLAX.

Observations of the celestial bodies such as the sun planets and moon when made from different parts of the earths surface, and corrected for semi-diameter—refraction do not give the same results, as owing to their comparatively small distances from us their position in the celestial sphere appears different to spectators situated at different points on the earths surface.

Hence astronomers in order to work on a common basis reduce all observations to the same point, viz.: the earth's centre. This reduction from the surface to the centre is called parallax; is always an additive quantity and may be defined as the angle subtended by the earth's radius for the place of the observer, and two lines drawn from the centre of the heavenly body under observation to the extremities of this radius.

When the body is in the zenith the parallax is nil, and is greatest when on the horizon; and the parallax in this latter case is called the horizontal parallax.



In the accompanying figure let O represent the position of an observer, and S that of a heavenly body, then according to our definition the angle O S C = P is the parallax, and when the body is on the horizon O S′ C is the horizontal parallax. Now in the △ O S C calling S O H the altitude above the horizon, we have

$$C O : C S = \sin. O S C : \sin. O S$$

that is calling C O radius earth $= r$, and C S distance of S $=$ R, and $\propto$ the apparent altitude.

$$r : R = \sin. P \cos. \propto$$

$$\text{or } \sin. P = \frac{r}{R} \cos. \propto \qquad (1)$$

when $\propto = 0$ we have in equation (1) P a maximum, and in that case if P be a small quantity,

$$\sin. P = P = \frac{r}{R} = \text{horizontal parallax.}$$

when $\propto = 90$ we have

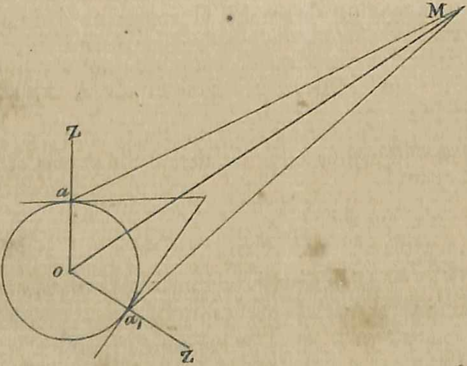
$$P = \frac{r}{R} \cos. 90^\circ = 0.$$

Now if the radius of the earth and the distance of the moon were constant quanties it is evident that the quantity $\frac{r}{R}$ would likewise be constant, but as the earth is an oblate spheroid, and the moon describes an ellipse round the earths surface, r will vary with the position of the observer, and R with that of moon, although in the case of the sun and the exterior planets, this variation in R will produce no appreciable effect in the horizontal parallax, the case is different for the moon which is at an average distance from the earth of only 60, the earth's radius. Therefore, in order to arrive at a fixed basis for the moon, tabulated values of the angle subtended by the equatorial radius of the earth is made out. The tabulated value is called the moon's horizontal equatorial parallax, and is given in page 3 of the *Nautical Almanack* for Greenwich noon and midnight for every day of the year.

From this tabulated value the horizontal parallax at any latitude on the earth's surface is obtained by decreasing the horizontal equatorial parallax for time of observation in the ratio (vide tables of Lomis) of the radius of the observer to the equatorial radius; then to obtain the parallax in altitude we proceed as above to multiply the sine of this value by cos. $\propto$, when we obtain the sine of the parallax in altitude. In the case of stars, of course there is no correction for parallax owing to their immense distance.

Now to illustrate what has been said about parallax as an aid to find distances.

Let M be the moon at any time, O the centre of the earth, a, a' two observatories on the same meridian. Let Oa and O M, the radius of the earth and moon's distance, be denoted by r and P respectively, and let Z a M and Z′ a' M, which are the moon's zenith distances, as seen from the two observatories, be denoted by Z and Z′. Then by equation (I)



$$\sin.\, p = \frac{r}{R} \sin.\, Z$$

$$\text{or } p = \frac{r}{R} \sin.\, Z$$

will be the parallax O M a of the moon at observatory a,

$$\text{and } p' = \frac{r}{R} \sin.\, Z$$

will be the parallax O M a' at observatory a'; adding these together we get

$$p + p' = \frac{r}{R} (\sin.\, Z + \sin.\, Z).$$

But the angle $p + p'$ or a M a' is equal to the difference between Z O Z′ and the sum of the angles Z and Z; and, since, if the observatories be situated one to the north and the other to the south of the equator, we have $Z\,O\,Z'$ equal to the sum of the latitudes of the two observatories $Z+Z'$, we obtain

$$p+p'=Z+Z'-l-l'$$

and substituting this value in preceding equation we get

$$Z+Z'-l-l' = \frac{r}{R}\ (\sin.\ Z + \sin.\ Z')$$

and solving for R we have

$$R = r \times \frac{\sin.\ Z + \sin.\ Z'}{Z + Z' - l - l'}$$

Now, we have seen that $\frac{r}{R}$ = horizontal parallax = P

$$\therefore P = \frac{r}{R} = \frac{Z + Z' - l - l'}{\sin.\ Z + \sin.\ Z'}$$

Now the moon's horizontal parallax p is found to be $57'\ 6''$, and since the number of seconds in any $\angle = 206265 \times \frac{\text{arc}}{\text{rad.}}$

we have $3426'' = 206265\ \frac{r}{R}$

hence, if r be taken at 3963 miles, the true distance is 237640 miles.

In the same way, the distance of the sun or planets can be found when horizontal parallax is known.

Now knowing the moon's distance we have only to observe its apparent diameter and use the following proportion :

Moon's diameter in miles : distance in miles :: Moon's diameter in seconds : number of seconds in an arc equal to radius.

The average diameter is $31'{\cdot}1''{\cdot}8$ and we get therefore

diameter : 236740 :: $1868''{\cdot}8$: 206265 ∴ diameter = 2153 miles.

The effects of parallax in right ascension and declination may be easily calculated, the method of doing so is given in most works on astronomy.

The Moon is about 240,000 miles off, or about 30 times the earth's diameter. Its apparent path is a great circle of the celestial sphere, inclined at an angle of about $5^\circ\ 8'\ 48''$ to the ecliptic. Its revolution, with reference to a star, is complete in 27d. 7h. 43m. Its *apparent* motion is from east to west, caused by the Earth's diurnal motion from west to east, but the Moon's *real* motion is from west to east also.

Since the Moon makes a complete revolution in 27d. 7h. 43m., its apparent motion is $13^\circ\ 10'\ 35''$ daily or $32'\ 56''$ per hour, *i.e.*, about its own diameter per hour.

Now the Sun makes a complete apparent circuit of the ecliptic in 365.25 days, and therefore moves over it at the rate of $61'$ *per diem.*

As the Sun and the Moon appear to move in the same direction (W to E), the Moon will, after conjuction (being in the same line as the Sun and Earth's centre), depart from the Sun at about the rate of $12^\circ{}'9$ per diem ($13^\circ\ 10'\ 35''-61'$). If then the Moon be in conjunction with the Sun, on any given day, it will be $12^\circ\ 9'$ east of it the next day, $24^\circ\ 18'$ east of it after two days and so on.

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If, therefore, the Moon set with the Sun on any evening it will be at sunset on the following evening 12° 9′ east of the Sun and appear as a thin crescent at a considerable altidude; on the suceeding evening it will be 24° 18 east of the Sun, at a still greater altitude; at sunset and a broader crescent. After seven days the Moon will be nearly 90° from the Sun, *i.e.*, at or near the meridian at sunset, and after about six hours will appear on the western horizon as a half-moon.

Each successive evening increasing its distance and its own breadth, after about 15 days it will be 180° from the Sun, and will be full, and consequently will rise as the Sun sets, being visible the entire night.

After about 22 days, the distance from the Sun being about 270°, the Moon will not reach the meridian until nearly sunrise, being visible only during the last six hours of the night, and so on until towards the close of the month, when the Moon will be seen in the morning shortly before sun-rise and then only as a thin crescent.

If the Earth and the Sun were both stationery while the Moon revolves round the former, the phases would be the same as the period of the Moon, but, as we have seen, while the Moon makes its apparent revolution of the heavens in about 27·3 days, the *Sun* advances through somewhat more than 27° of the heavens in the *same direction.*

Before the Moon can reassume the same phase, it must have the same position relatively to the Sun, and must therefore *overtake* it, and since it moves at the rate of about 1° in two hours (12° 9′ daily) it will take more than two days to move over 27°; hence the synodic period or lunar month, or the interval between two successive conjunctions is about two days longer than the sidereal period of the Moon. The *exact* length of the synodic is 29·53059 mean solar days.

Now having run over the motions of the Sun and Moon among the stars, let us consider the *relative* positions of the stars themselves so as to recognize them when required.

FIXED STARS IN OR NEAR THE ZODIAC.

Suppose we commence our observations about the middle of January in the latitude of Chatham. About 9 o'clock in the evening, if we look towards the south, Orien will be seen a little to the east of the meridian. It is distinguished by four bright stars in the form of an oblong, and more particularly by three brilliant stars *Orion's Belt* in the centre of the oblong. Two only of the stars in this constellation are of the 1st magnitude: *Betelgueux* on the east shoulder, and *Rigel* in the left foot on the west; a line through the uppermost star in the Belt will connect the two (19° distant from each other). Bellatrix in the west shoulder (2nd magnitude) is distant from Betelgueux about 7½°, 10° SW from the same star, lie three stars forming the Belt, nearly in a line with each other.

In a SW direction distant about 19° from Betelgueux is *Rigel* (1st mag.) in the left foot of Orion. The few small stars running in a S direction from the Belt are called the *Sword* of Orion, and about 5° in a N.W.

direction and situated halfway between Betelgueux and Bellatrix is a small cluster of stars called *Orion's Head.*

The same line continued will pass through *Aldebaran* about 14° off: a bright red looking star (1st mag.), which, with others stars in the Bull's Head (Taurus,) make the figure V; this line carried on will point out the *Pleiades*, a remarkable cluster of numerous small stars, seven only visible to the unnasisted eye. They are about 14° distant from *Aldebaran.*

West of the Pleiades, distant about $23\frac{1}{2}$°, is a *Arietis* (2nd mag.) in *Aries* the Ram, and S. W., about 23° is *Menkar* in Cetus. The three (Pleiades, a Arietis and Menkar) form nearly an equilateral triangle.

A line from the Pleiades, through the Belt in Orion, and about 23° from the middle star, will pass through *Sirius* (1st mag.) in Canis Major; and about 5° S.W. is β in Canis Major (2nd mag.).

North east from Sirius, distant about 25°, is *Procyon* (1st mag.) in Canis Minor

Betelgueux, Procyon, and Sirius, form an equilateral triangle, and being all brilliant stars are easily distinguishable.

A line from Procyon, carried about $22\frac{1}{2}$° N. will pass through *Pollux* (2nd mag.), and 5° N. W. of Pollux is *Castor* (1st mag.); both in the constellation Gemini.

A line from Castor through the middle star of the Belt of Orion will lead to Rigel, 53° off Castor

A line carried N. W. from the middle star of Orion's Belt through Orion's Head will pass through β *Tauri*, about $29\frac{1}{2}$° off (2nd mag.); the same line produced 18° further will pass over *Capella*, a beautiful star (1st mag) in Auriga.

About 10° E. N. E. from Procyon, in a line with Betelgueux and Bellatrix is β *Cancri* (3rd mag.) which linealso passes through β *Canis Majoris*, about $22\frac{1}{2}$° from Betelgueux.

A line from Betelgueux, about $1\frac{1}{2}$° north of β in Canis Minor, will pass over the beautiful star *Regulus* (1st mag.) in Leo.

S. S. W., $22\frac{1}{2}$° from Regulus is *Cor Hydræ* (2nd mag).

A line from Procyon, through Regulus, will pass over *Denebola*, or β Leonis, (1st mag.), about $24\frac{1}{2}$° from Regulus, and 14° N. E. from Denebola, is a small cluster of stars called *Coma Berenices.*

E. by S. from Denebola is *Vindemiatrix* (3rd mag.), in Virgo, about $17\frac{1}{2}$° off.

A line, S. E from Denebola, 36° off, will reach *Spica*, Virginis (1st.).

A line from Regulus through Denebola, and 36° from the latter, will reach *Arcturus* (1st mag.) in Bootes.

21° S.E. from Spica is *Eubenesch* (2nd mag.), in Libra, and N. E. $8\frac{1}{2}$° in *Gubenelg* (3rd mag.), in the same constellation.

A line from Spica, carried a little to the S. of *Zubenesch* and produced about 47°, will cross *Antares* (2nd Mag.), the largest star in Scorpio.

These last three form a right angled triangle, Zubenelg being at the right angle.

10° N.E. from Arcturus is *Mirach*, and a line from Vindemiatrix through Arcturus will pass *Alphacca* about 21° off (2nd mag.) in Coronœ Borealis, or the Northern Crown.

A line from Alphacca in a S.E. by E. direction, 27° off, will pass through *Ras Algethi* (2nd mag.) in Hercules, and point out *Ras Alhague* in Ophiuchi, about 6°.

N.B.—This star and Regulus are very advantageously observed on the meridian in connection with Polaris off the meridian for latitude.

A line from this latter star E. by S. 33° will pass through *Altair* in Aquila the Eagle.

(This star is on the meridian about 10 o'clock on the evening of the 1st September; it is between 1st and 2nd magnitude, and has a small star to the north and a small star to the south of it, of the 4th and 3rd magnitudes).

A line N.W., about 34° from Altair, will cross V*ega* or α Lyræ (1st mag.). This star, with Ras Alhague and Altair, form nearly an equilateral triangle. S. by E., 22° from Altair is α *Capricorni*, and the line produced 3° in the same direction will cross β *Capricorni*, both 3rd magnitude.

A line from Altair S.E., produced about 60°, will point out *Fomalhaut* (1st mag.) in *Pisces Australis*, being one of few stars in the Southern Hemisphere of the 1st magnitude.

E.N.E. from Altair is the constellation Delphinus, formed of four bright stars close together in a diamond shape, the centre of the diamond being about 14° from Altair; a line from which star through the southern one of Delphinus will pass over *Scheat* or β Pegasi (1st mag.), at a distance of about 50°, and point out *Mirach*, or β in Andromeda of the 2nd magnitude, distant from Scheat about 32°.

About 15° E. from Scheat is Alpheratz, in the same constellation. 13° S., a little E., from Alpheratz is Algenib or γ in Pegasus; and 18° due W. from Algenib is Markab or α in Pegasus. These four stars, all of the second magnitude, form a square, which will be a little to the E. of the meridian about the middle of October, at 2 o'clock in the evening.

THE NORTHERN HEMISPHERE.

Ursa Major will be the best constellation to begin with, as every one knows it.

Of the four bright stars in the square, the two most distant from the tail are called the "Pointers," from their pointing to the pole star, a star between the 2nd and 3rd magnitudes, in the tail of Ursa Minor, and from which *Dubhe* of the 1st magnitude, is distant about 28°; Dubhe is one pointer, the other is called β *Ursæ Majoris*, and the three stars in the tail of the Great Bear are, Alioth, Mizar, and Benetnasch, the latter being the extreme point of the tail.

The Pole Star is about 1½° from the pole.

A line through γ and δ Ursæ Majoris, (the two stars which make up with the pointers) carried about 15° will pass through α *Draconis* (2nd mag.); and a line from Mizar, the middle star in the tail of the Great

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Bear, carried through α Draconis will pass over the two upper stars in the square of the Little Bear.

A line from α Draconis, carried through Mizar, will point out *Cor Caroli* in Canes Venatici, at a distance of 17°.

A line from Benetnasch through Cor Caroli will terminate at about 29° off in *Denebola* (β Leonis).

Cor Caroli, Antares in Scorpio, and Alphacca in Coronæ Borealis form a right angled triangle, right-angled at Antares.

A line from Dubhe about 2° W. of β in Ursa Major will point out *Regulus* α Leonis.

From Alioth, a line carried through the Pole Star, will lead to γ in *Cassiopeia*, about 28° off.

In Cassiopeiæ the form of the five stars represents a W or a chair; the farthest from the pole is called *Schedir*.

Nearly midway between α *Draconis* and *Vega* in Lyra is a small cluster of stars in Draco, the two principal ones are *Rastaban*, nearest to Draco, and *Etanin*, about 15° from Vega.

About 26° E.N.E. from Vega is *Adrided* in Cygnus (1st & 2nd mag). The four principal stars in this constellation form a large cross, and *Vega*, *Etanin*, and *Adrided* form rectangular triangle, right angled at Vega.

A line from *Algenib* in Pegasus through *Scheat* will lead to *Adrided* also.

From Schedir in Cassiopeiæ a line through β Cassiopeiæ, and produced about 25°, will pass about 2° N. of *Alderamin* in Cepheus (3rd mag.). The same line carried 20° in the opposite dircction from Schedir will terminate in *Almaach*, a star of the 2nd magnitude, in Andromeda.

A line from the Pole Star, produced through β Cassiopeiæ a distance of 30° will pass over *Alpheratz*. These three stars will come on the meridian about the same time.

Mirach or β Andromedæ is nearly midway between *Alpheratz* and *Almaach*, distance from the latter about 14°, from the former about 17°.

A line from *Mirach* through *Almanack* and produced beyond it 17° will cross *Algenib* in Perseus. The two small stars near Algonib make, with it, what is called the Bow of Perseus.

Algol, the chief star in Caput Medusæ, lies about 10° S. of *Algenib* and forms nearly a right angled triangle with it and *Almaach*, the right angle being at *Algol*. [This star Algol is a variable star, and varies from the 1st to the 4th magnitude in about 2d. 20h. 49m.] It is situated about 12° E. of *Almaach* in the foot of Andromeda. It comes on the meridian about 9 o'clock in the evening in the third week of December. A line from *Almaach*, passing midway between *Algenib* and *Algol* and produced about 38°, will pass over *Capella* in Aurigan the same line continued to about 32°, will pass at a little to the south of β in *Auriga* and terminate in *Castor* in Gemini. A small triagle; lies to the south of Capella, bordering on the Milky Way.

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S.E. by S., $4\frac{1}{2}$° from *Castor* (α Geminorum) 1st magnitude, is Pollux or β *Geminorum* (2nd magniture).

THE SOUTHERN HEMISPHERE.

Comparatively few remarkable stars are to be seen in the Southern Hemisphere.

The most striking constellation is the *Southern Cross*, consisting of four bright stars, one only (the southernmost) being of the 1st magnitude. They are called α, β, γ, δ; α being the Southern Star (1st mag.), β the star on the east (2nd mag.), γ the Northern Star (2nd mag.), and δ the west star (3rd mag.).

A line from *Sirius* through γ in *Canis Major* will pass over ζ in *Argo Navis*, and point out γ in *Crux*, the Cross, at a distance of 46° from ζ in *Argo*.

In a south direction and about 36° from *Sirius* will be found *Canopus*, or α in *Argo Navis*, (1st magnitude).

A line from α *Crucis* to *Canopus* will leave β in *Argo Navis* a little to the north. This star is of the 1st magnitude, and is about 25° from *Canopus*. This line continued in a northerly direction will pass over (at a distance of 20°) α *Columbæ* (2nd mag.), and produced 28° in the same direction will terminate in *Rigel* (left foot of Orion). *Rigel Sirius* and α *Columbæ* form nearly an equilateral triangle.

18° from *Canopus* and E. by N. from it is γ in *Argo Navis* (2nd mag.). This star is the middle one of the three of the same magnitude in this constellation: the one 12° to the south of it is called ε, and the one 8° to the north ζ.

A line from δ in *The Cross*, carried through β in the same constellation, will pass over β *Centauri* at a distance of 9°, and produced 5° further will terminate in α *Centauri* (both 2nd mag.).

42° S.E. by E. from α *Centauri* is the star α in *Pavo*, the Peacock. This star lies in a southerly direction from *Antares* about 51° off.

A line E.N.E. from α in *Pavo* will pass over α *Crucis* at the distance of 18°, and carried 20° further, nearly in the same direction, will terminate in Fomalhaut or α in the Southern Fish. This is a brilliant star (1st mag.) and stands almost alone in this part of the heavens.

41° in a south-easterly direction from α Pavonis is α Eridani or Achernar (1st mag.). These stars, with Fomalhaut, form an equilateral triangle.

Nearly midway between Fomalhaut and Achernar and about 20° to the N.W. of the latter is α *Phœnicis* (2nd mag.).

N.B.—*Distances* may be accurately measured by the eye when once accustomed to estimate rightly any known distance—as, for instance the space between the two pointers, which is about 5°; and that again between Betelgueux and Bellatrix, which is nearly 8°.

TIME.

BY AN ALTITUDE OF THE SUN—THE LATITUDE BEING KNOWN.

Morning Observations.—If we take an altitude of the sun when rising,

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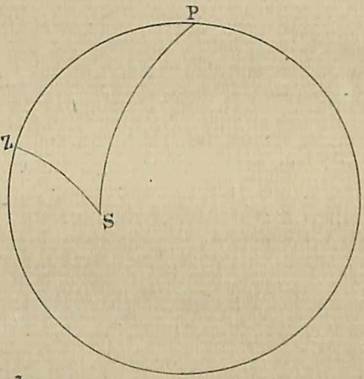
i.e., increasing in altitude, and reduce it to the true altitude of its centre, and knowing our latitude and the sun's declination at the time of observation we evidently get by taking the complements of the altitude, declination and latitude (or 90° + dec. if south) the 3 sides of the astronomical triangle as shown in diagram, viz. :—



P Z = co. latitude.

P S = polar distance = (90° ± dec.) according as dec. is south or north.

Z S = co. altitude (zenith distance).

To solve for P the hour angle.

Having obtained this by the solution of this triangle by formula

$$\cos. \frac{P}{2} = \sqrt{\frac{\sin. S \sin. (S - Z S)}{\sin. Z P \sin. P S}}$$

$$\text{or } \sin. \frac{P}{2} = \sqrt{\frac{\sin. (S - Z P) \sin. (S - P S)}{\sin. Z P \sin. P S}}$$

We get the distance in degrees, minutes and seconds, of the sun's centre from the meridian, which when converted into hours, minutes, and seconds at the rate of 360° in 24 hours, gives the *interval* in time before apparent noon; if this interval be subtracted from 12, the result is apparent time, and applying the equation of time, as found from the *Nautical Almanack* (corrected of course for time of observation and longitude,) we obtain the local mean time of observation, and comparing this with the mean time noted by our chronometer, we find its error, and successive observations day by day will give us its *rate* of gaining or losing, so that for any observation subsequently taken, the actual time of observation is known by the simple application of the known rate.

Afternoon Observations.—If the sun has crossed the meridian and is decreasing in altitude, as shown in the figure at S^1, the solution of the triangle Z P S^1 for P and converting into hours, minutes, and seconds, and seconds gives us the interval in time from the meridian, *i.e.*,

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apparent time at once, and the application of the equation of time, as before, gives us the local mean time of observation. The nearer the sun is to the prime vertical for these observations the better.

BY AN ALTITUDE OF A STAR.

East Star.—In the previous figure if we suppose the altitude of S any star rising rapidly in the eastward has been observed, and that as before we solve for P the hour angle and turn it into h. m. s. we evidently obtain the interval (in sidereal time) from the sidereal time of the meridian passage, now the sidereal time at the meridian passage is=the right ascension of the star, so if we take the R A of the star from the *Nautical Almanack*, and subtract the interval found, we obtain the sidereal time of observation, and knowing the sidereal time at the preceding mean noon and substracting this from the sidereal time of observation, adding 24 hours if necessary, we get the interval past preceding mean noon in sidereal time; and if we convert this interval into mean time by the table of mean equivalents, we have the interval past preceding mean moon, in mean time.

This interval compared with the interval as shown by our chronometer time of observation will give us the error of our chronometer in mean solar time.

West Star.—In the previous figure, if we suppose the altitude of S^1 any star setting rapidly to the westward, has been observed, and, as before the interval from the meridian in sidereal time obtained, all we have to do is to add the sidereal time of the meridian passage (=R A by def^{n}.) to the interval, to obtain the sidereal time of observation. The difference between this and the sidereal time of mean noon at place of observation gives the sidereal time since mean noon, which converted into its mean equivalent gives mean time at place, and hence the error of the chronometer.

As the sidereal time of mean noon must evidently be always increasing from day to day, the correction for longitude (·0027379 × No. of seconds E. or W. of Greenwich,) must be subtracted if the longitude be east, from the sidereal time at mean noon at Greenwich to obtain sidereal time at mean noon at place of observation—or added if longitude be west.

(Of course if the time of observation be noted by a sidereal clock, the difference between the time noted and the R A of the star ± the interval found by solving for the hour angle P, gives at once the error of the clock in sidereal time.)

BY EQUAL ALTITUDES OF A STAR ON OPPOSITE SIDES OF THE MERIDIAN.

If we take a series of altitudes of a star, (say at 70° — 70° 10′ — 70° 20′ and so on,) when rising rapidly in the east, and wait until it descends to the same series of altitudes to the west, noting in reversed order the times of observation, the arithmetical mean between the means of the two sets of times is the time of the star's meridian passage, and we have only to compare this with the right ascension of the star as known *(Nautical Almanack)* to obtain the error of the clock.

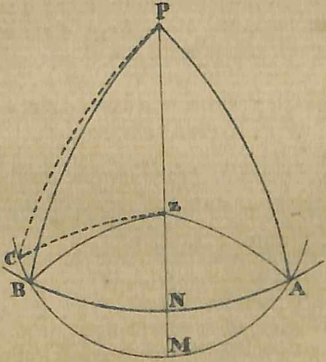
Or if we use a mean time chronometer, we must convert the R A of the star, (*i.e.*, the sidereal time of its meridian passage,) into mean time before comparing it with the arithmetical mean of the times as noted by the chronometer.

It will be evident that this method must be susceptible of far greater accuracy than that by the single altitude, as the effects of refraction and index error (assuming that the temperature or barometric pressure does not change very considerably in the interval) do not affect the truth of the conclusion—the times of the star having *equal* altitudes east and west being all that we want, apart altogether from the altitudes taken being the true altitudes at the moment or not.

BY EQUAL ALTITUDES OF THE SUN ON OPPOSITE SIDES OF THE MERIDIAN.

If we take a series of altitudes of the sun in the same way as we have supposed in the case of the star in the preceding paragraph, the mean of the times observed will not be *quite* the time of the sun's meridian passage. For this reason:—The declination of the sun changes constantly, and the time of its arrival at a given altitude will be affected by this motion, and it becomes, therefore, necessary to compute the correction required.

In the figure, let P Z M be the meridian of the place of observation, P the pole, Z the zenith, A M B a small circle parallel to the horizon, A N B the parallel described by a star in its diurnal motion, and cutting A M B in A and B.



If Z A be found by observation equal to Z B, then since P Z (the co. latitude) is constant, if the polar distance be constant (as in the case of a star) the two triangles P Z A, P Z B the three sides, will be mutually equal and consequently the angle Z P A = Z P B, *i.e.*, the hour angle of a star from the meridian is the same for equal altitudes on the east and west sides of the meridian.

As we have seen, however, the sun's polar distance *changes*. Now suppose the polar distance of the sun has diminished during the interval, then, when the western hour angle Z P B is equal to the eastern hour angle Z P B, the sun will be at B^1, *i.e.*, *nearer the zenith*, and when the sun reaches the circle A M B at C, the hour angle Z P C will be greater than the hour angle Z P A or Z P B.

It is necessary, therefore, to compute B P C, the hour angle of difference.

Let $\phi =$ latitude of place; $\delta =$ the declination of the sun when on the meridian; $d\,\delta =$ the increase of declination (or decrease of polar distance) from the meridian to the afternoon observation; $\mathrm{P} =$ hour angle from the meridian, supposing no change in declination; $d\,\mathrm{P} =$ the increase of the hour angle caused by the change in declination; and $z =$ observed zenith distance.

Then we have in the $\triangle$ A P Z.

$$\text{Cos. AZ} = \text{cos. PZ cos. AP} + \text{sin. PZ sin. AP cos. APZ}$$

$$\text{Or cos. } z = \text{sin. } \phi \text{ sin. } \delta + \text{cos. } \phi \text{ cos. } \delta \text{ cos. P} \ldots\ldots \text{(I.)}$$

And in the $\triangle$ C P Z.

$$\text{Cos. CZ} = \text{cos. PZ cos. CP} + \text{sin. PZ sin. CP cos. CPZ}$$

$$\begin{aligned}\text{Or cos. } z &= \text{sin. } \phi \text{ sin. } (\delta + d\,\delta) + \text{cos. } \phi \text{ cos. } (\delta + d\,\delta) \text{ cos. } (\mathrm{P} + d\,\mathrm{P}) \\ &= \text{sin. } \phi \text{ sin. } \delta \cos d\,\delta + \text{sin. } \phi \text{ cos. } \delta \text{ sin. } d\,\delta \\ &+ \text{cos. } \phi\, (\text{cos. } \delta \text{ cos. } d\,\delta - \text{sin. } \delta \sin d\,\delta)\, (\text{cos. P} \cos d\,\mathrm{P} - \text{sin. P sin. } d\,\mathrm{P})\end{aligned}$$

And since the variations of δ and P are very small we may put

$\text{Cos. } d\,\delta = 1$; $\text{cos. } d\,\mathrm{P} = 1$; $\text{sin. } d\,\delta = d\,\delta \text{ sin. } 1''$ $\text{sin. } d\,\mathrm{P} = 15\, d\,\mathrm{P} \text{ sin. } 1''$

$$\begin{aligned}\therefore \text{Cos. Z} = {} & \text{sin. } \phi \text{ sin. } d + d\,\delta \text{ sin. } 1'' \text{ sin. } \phi \text{ cos. } \delta + \text{cos. } \phi \text{ cos. } \delta \text{ cos. P.} \\ & - d\,\delta \text{ sin. } 1'' \text{ cos. } \phi \text{ sin. } \delta \text{ cos. P} \\ & - 15\, d\,\mathrm{P} \text{ sin. } 1'' \text{ cos. } \phi \text{ cos. } \delta \text{ sin. P.}\end{aligned}$$

Subtracting equation I from this, and dividing by sin. 1″

$$\begin{aligned}0 = {} & d\,\delta \text{ sin. } \phi \text{ cos. } \delta - d\,\delta \text{ cos. } \phi \text{ sin. } \delta \text{ cos, P} \\ & - 15\, d\,\mathrm{P} \text{ cos. } \phi \text{ cos. } \delta \text{ sin. P.}\end{aligned}$$

Whence

$$\begin{aligned}d\,\mathrm{P} &= \frac{d\,\delta \text{ sin. } \phi \text{ cos. } \delta - d\,\delta \text{ cos. } \phi \text{ sin. } \delta \text{ cos. P}}{15 \text{ cos. } \phi \text{ cos. } \delta \text{ sin. P.}} \\ &= \frac{d\,\delta}{15} \cdot (\text{tan. } \phi \text{ cosec. P} - \text{tan. } \delta \text{ cot. P}) \ldots\ldots\ldots\end{aligned}$$

Which is the correction to be applied to the mean of the times observed.

N.B.—If the sun's motion in declination be towards the elevated pole, this correction is to be subtracted from the mean of the times observed; if the motion be away from the elevated pole, it must be added. The result gives the clock time of apparent noon, which compared with the mean time of apparent noon as obtained from *Nautical Almanack*, gives the error of chronometer.

The above equation, which is called the Equation of Equal Altitudes, can be obtained more easily by differentiating the equation $\text{cos. P} = \dfrac{\text{cos. } z - \text{sin. } \phi \sin \delta}{\text{cos. } \phi \text{ cos. } \delta}$, on the supposition that δ and P vary. We have

$$\text{cos. } z = \text{cos. P cos. } \phi \text{ cos. } \delta + \text{sin. } \phi \text{ sin. } \delta.$$

$$\text{differentiating } 0 = \text{sin. } \phi \text{ cos. } \delta\, d\,\delta - \text{cos. } \phi \text{ sin. } \delta\, d\,\delta \text{ cos. P} - \text{cos. } \phi \text{ cos. } \delta \text{ sin. P}\, d\,\mathrm{P} = 0.$$

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$$\text{and therefore } d\,\mathrm{P} = d\,\delta\left(\frac{\sin.\phi\cos.\delta - \cos.\phi\sin.\delta\cos.\mathrm{P}}{\cos.\phi\cos.\delta\sin.\mathrm{P}}\right)$$

$$= d\,\delta\,(\tan.\phi\ \mathrm{cosec.\,P} - \tan.\delta\cot.\mathrm{P}), \text{ and in seconds of time.}$$

$$d\,\mathrm{P} = \frac{d\,\delta}{15}(\tan.\phi\ \mathrm{cosec.\,P} - \tan.\delta\cot.\mathrm{P})$$

as above.

Example.—At a place, latitude 54° 20′ N. the sun was found to have equal altitudes at 8h. 59m. 4s. a.m. at 3h. 0m. 40s. p.m.

It is required to find the time of noon, the declination being 19° 48′ 29″, and the decrease of declination between the two observations being 192″.

By equation (II.)

$$\begin{array}{rlcrl}
\tan\phi = & 0{\cdot}14406 & & \tan.\delta = & 9{\cdot}55652 \\
\mathrm{Cosec\ P} = & 0{\cdot}14900 & \text{and,} & \cot.\mathrm{P} = & 9{\cdot}99697 \\
\frac{d\,\delta}{15} = 6{\cdot}4 = & 0{\cdot}80618 & & \frac{d\,\delta}{15} = 6{\cdot}4 = & 0{\cdot}80618 \\
12{\cdot}57\mathrm{s.} = & 1{\cdot}09924 & & 2{\cdot}29\mathrm{s.} = & 0{\cdot}35967
\end{array}$$

$$\therefore d\,\mathrm{P} = 12{\cdot}57 - 2{\cdot}29 = 10{\cdot}28\mathrm{s.}$$

To be added to the mean of times observed, as the sun's motion is southward.

The mean of the observed time is 11h. 59m. 52s.

∴ The clock time of apparent noon = 11h. 59m. 52s. + 10·28s.
= 12h. 0m. 2·28s.

or the chronometer was too fast by 2·28s. on apparent time.

In order to facilitate this computation, the correction may be obtained in the following manner.

Let T = the interval of time between the morning and afternoon observations, expressed in hours.

μ = hourly change of the sun's declination.

Then since $d\,\delta$ represents the increase of declination from the meridian to the afternoon observation, we shall have

$$d\,\delta = \tfrac{1}{2}\mu\,\mathrm{T}$$

and since P represents the hour angle from the meridian expressed in arc, we have

$$\mathrm{P} = 7\tfrac{1}{2}\,\mathrm{T}.$$

Hence the correction to be added to the mean of the times observed to obtain the time of apparent noon is

$$x = -\frac{\mu\,\mathrm{T}\tan.\phi}{30\sin.7\frac{1}{2}\mathrm{T}} + \frac{\mu\,\mathrm{T}\tan.\delta}{30\tan.7\frac{1}{2}\mathrm{T}}$$

$$\text{or } x = -\mu\tan.\phi\frac{\mathrm{T}}{30\sin.7\frac{1}{2}\mathrm{T}} + \mu\tan\delta\frac{\mathrm{T}}{30\tan.7\frac{1}{2}\mathrm{T}}$$

$$\text{And calling } \mathrm{A} = \frac{\mathrm{T}}{30\sin.7\frac{1}{2}\mathrm{T}}$$

$$\text{And } \mathrm{B} = \frac{\mathrm{T}}{30\tan.7\frac{1}{2}\mathrm{T}}$$

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we have $x = -\text{A}\tan.\phi + \text{B}\,\mu\tan.\delta$.

Now to compute A and B

Let T = 2 hours.

$$\begin{array}{rlrl}
\text{Then T} \times 7\tfrac{1}{2} = 15^\circ \sin. = & 9{\cdot}4130 & \text{T} \times 7\tfrac{1}{2} = 15^\circ \tan. = & 9{\cdot}4280 \\
\text{Log. } 30 = & 1{\cdot}4771 & \text{Log. } 30 = & 1{\cdot}4771 \\
\hline
 & 0{\cdot}8901 & & 0{\cdot}9051 \\
\text{Log. } 2 = & 0{\cdot}3010 & \text{Log. } 2 = & 0{\cdot}3010 \\
\hline
\text{Log. A.} = & 9{\cdot}4109 & \text{Log. B.} = & 9{\cdot}3959
\end{array}$$

So the value of other members may be computed and tabulated, as they are in Loonis and other practical works.

Now suppose in the example of equal altitudes used before, we apply these tables.

The interval in time between the morning and afternoon observations being 6h. 1m. 36s., we have from the tables Log. A $= 9{\cdot}4520$, and Log. B $= 9{\cdot}2999$, and from the *Nautical Almanack* $\mu = -31{\cdot}85''$, the correction is obtained as follows:—

$$\begin{array}{rlrl}
\text{Log. A} = & 9{\cdot}4520 & \text{Log. B} = & 9.2999 \\
-31{\cdot}85'' = \mu \quad = & 1{\cdot}5031\text{neg}^{\text{ve}} & -31{\cdot}85'' = \mu = & 1{\cdot}5031\text{neg}^{\text{ve}} \\
\text{Tan. } \phi = & 0{\cdot}1441 & \text{Tan. } \delta = & 9{\cdot}5565 \\
\hline
-12{\cdot}57\text{s.} = & 1{\cdot}0291\text{neg}^{\text{ve}} & -2{\cdot}22\text{s.} = & 0{\cdot}3595\text{neg}^{\text{ve}}
\end{array}$$

$\therefore x = 12{\cdot}57$s. $- 2{\cdot}29 = 10{\cdot}28$ the same correction as found before.

N.B.—The sign of the first term is positive from the summer to the winter solstice, and negative from the winter to the summer solstice.

The sign of the second term is positive from the equinoxes to the solstices, and negative from the solstices to the equinoxes.

The time may also be found by the altitudes of the sun or a star, and the elapsed time; or by the simultaneous observations of two stars.

This method will be available when the latitude is not known as explained under the head latitude. Proceeding as in para. page , we determine the latitude and the angle P S Z′. This angle with the colat. P Z, and the zenith distance Z S will determine the hour angle S P Z.

$$\text{Sin. Z} = \frac{\text{sin. Z S sin. Z P S}}{\text{sin. P Z}}$$

EFFECT OF ERRORS OF DATA ON THE TIME.

$$\text{Cos. P} = \frac{\sin. z - \sin. d \sin. \phi}{\cos. d \cos. \phi}$$

differentiating this equation on the supposition that P Z and ϕ are variables, we have—substituting for P its value in terms of A, the azmiuth,

$$d\,z + \cos.\text{A}\,d\,\phi = \cos.\phi\,\sin.\text{A}\,d\,\text{P}, \text{ where A is the zenith.}$$

or $d\,\text{P} = \dfrac{d\,z + \cos.\text{A}\,d\,\phi}{\cos.\phi\,\sin.\text{A}}$, or the error in time due to a small error in the latitude and altitude, and d P is in min. for A $= 90$ and a maximum for A $= 0$, and hence the best time to take an observation

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for time is when the body on which the observation is taken is due E. or W. of the observer, and the worst when the body is on the meridian. This rule, however, must not be intended to apply when the body is on the prime vertical near horizon. If $d\,\phi = 0$, or the latitude is accurately known, then

$$d\,\mathrm{P} = \frac{d\,z}{\cos.\,\phi\,\sin.\,\mathrm{A}}, \text{ in seconds of arc.}$$

$$\text{also } d\,\mathrm{P}' = \frac{d\,z}{\cos.\,\phi\,\sin.\,\mathrm{A}'}\; d\,\mathrm{P} : d\,\mathrm{P}' = \frac{1}{\sin.\,\mathrm{A}} : \frac{1}{\sin.\,\mathrm{A}'}$$

if A and A′ be 120, and 150 respectively, we have

$$d\,\mathrm{P} : d\,\mathrm{P}' = \frac{1}{\frac{\sqrt{3}}{2}} : \frac{1}{\frac{1}{2}} = \frac{1}{\sqrt{3}} :$$

$\therefore\ d\,\mathrm{P}' = d\,\mathrm{P}\,\sqrt{3}$, shewing that the effect of a constant error in altitude increases as the star approaches the meridian.

By differentiating the equation given above for cos. P, on the supposition that all the quantities are constant except the declination, and substituting we find at once the effect of a small error in the declination on the time. This also is least when the body is on the prime vertical. The time found by equal altitudes of a fixed star is quite independent of errors, in latitude and declination since these quantities do not enter into the computation. In observations of the sun we know that the equation of equal altitudes is

$\frac{d\,\delta}{15}\,(\tan.\,\phi\,\mathrm{cosec.}\,\mathrm{P} - \tan.\,\delta\,\cot.\,\mathrm{P})$ in this equation an error in the latitude effects the term

$$\frac{d\,\delta}{15}\,\tan.\,\phi\,\mathrm{cosec.}\,\mathrm{P} = a$$

and differentiating we have

$$\frac{d\,\delta}{15}\,\mathrm{cosec.}\,\mathrm{P}\,\sec.^2\,\phi\,d\,\phi = d\,a$$

in a similar manner we find that an error in the declination produces an error in the 2nd term $(= b)$.

$$= d\,b = \frac{d\,\delta}{15}\,\sec.^2\,\delta\,d\,\delta\,\cot.\,\mathrm{P}$$

However, as the latitude is always known within one minute more or less, and the declination within a few seconds, we may regard the method as pratically free from any errors in these quantities. The accuracy of the result will therefore depend wholly on the observations, and as we have already seen the value of the error in time due to an error in altitude, $d\,z$

$$\text{will be } d\,\mathrm{P} = \frac{d\,z}{30\,\cos.\,\phi\,\sin.\,\mathrm{A}} \text{ in seconds of time.}$$

LATITUDE.

The methods of determining the latitude may be classed under four heads—1st, meridian observations, 2nd, ex-meridian observations, 3rd, circum-meridian observations, 4th, by transits in prime vertical. The simplest method is by taking the meridian altitude of the sun or of a star. The sun is on the meridian at apparent noon, which is 12 mean time + or — the equation of time, this latter is given for each day in the *Nautical Almanack*, therefore if we take a meridian altitude at this moment, of the sun's upper or lower limb, we can by applying the correction for index error, refraction, semi-diameter, and parallax find the true altitude of its centre, and adding to or subtracting from this the declination, corrected for time of observation, we find at once the latitude as hereunder :—

Latitude = zenith distance + declination, if the sun be north of the equator, *i.e.*, from 21st March to 22nd (about) September ; or

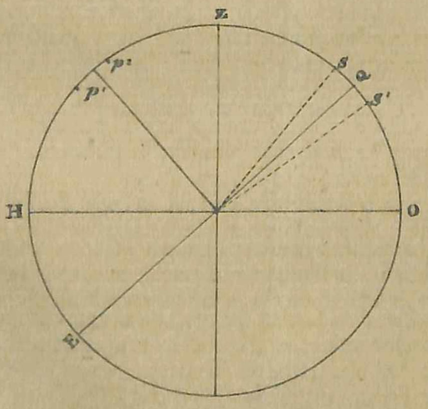
Latitude = zenith distance — declination, if the sun be south of the equator, *i.e.*, from the 23rd September to 21st March (about).

In this method it is to be observed that the meridian is not the greatest altitude in the case of the sun, moon, or a planet, but the meridian altitude may be obtained from the greatest altitude by means of the equations,

$$d = (9{\cdot}406 \times \frac{\Delta\,\delta}{\mathrm{A}} \qquad (1)$$

$$\text{and } y = \mathrm{A}\,\frac{2\sin.^2\tfrac{1}{2}d}{\sin.\,1''} \quad (2) \quad \text{when } \mathrm{A} = \frac{\sin.\,\phi\cos.\,\delta}{\sin.\,z}$$

by the first of these equations we get the interval in time between greatest altitude and meridian altitude, knowing the change of declination per minute ; by the second we obtain the correction to be applied to the greatest altitude to obtain the meridian altitude.



It is evident from the figure that if S or S′ (the sun or a star) be on the meridian P Z O, and the altitude S O, or S′ O above the horizon, (or a double altitude with an articficial horizon) be taken at that instant, the complements of these altitudes give us the distances S Z and S′ Z from the zenith (Z); and since S Q and S′ Q are (by definition) the declination North and South, we have

$$Z\,S + S\,Q = Z\,Q = \text{latitude.}$$

$$\text{and } Z\,S' - S'\,Q = Z\,Q = \text{latitude.}$$

[It may not be out of place just to draw attention to the fact that the latitude of the place (Z Q) is equal to the altitude of the pole (P H), both being equal to 90° minus the same arc P Z, which at the same time is evidently the co-latitude].

LATITUDE BY ALTITUDE OF CIRCUMPOLAR STARS AT TIME OF CULMINATION.

It must be remembered that the Pole Star moves apparently in a small circle about $1'' 23'$ from the actual pole, so that an altitude of Polaris even on the meridian will not give the latitude of the place, but $1^\circ 23'$ (nearly) too much, or too little according as it is at the upper or lower culmination. But it will be evident that if we take an altitude of Polaris or any circumpolar star at its upper *and* 12 hours after at its lower culmination, *half* the sum of the altitudes (corrected for instrumental errors and refraction of course) will be the altitude of the actual pole, and, therefore, the latitude of the place.

It has been mentioned that combining observations, so as to get rid of refraction and index error, is a great advantage; and, while talking of Polaris, we will just see how we can apply this suggestion.

Suppose we take altitudes of Polaris at its upper and lower culminations at p_1 and p_2, we obtain the altitude of P the actual pole, or we should do so if there were no such thing as refraction and if our instruments were perfect and free from index error.

This not being the case however, suppose our index error to be (ϵ) + or − as the case may be.

Owing to refraction, the altitudes taken of p_1, p_2 will be both too large, and ∴ (apart from index error) the altitude of the actual pole (P H) will be too large also, and suppose it to be P′ H as shown in at (P′).

That is, the zenith distance we get will be *too small* by the amount due to refraction. Suppose, however, we turn round and take the altitude of some star on the meridian southward of the zenith, as at S, of the same altitude, or nearly so, as Polaris, the effect of refraction will give us our zenith distance S Z *too small* again, by nearly the same quantity. Now, looking upon P′ as a star on the meridian between the zenith and the pole, and calling A its observed altitude and r the refraction + ϵ the I.E., then the true altitude is $A + \epsilon - r$; the polar distance is equal to $90 - d$ and the latitude is

$$\epsilon + A - r - (90 - d) = \text{lat.} \qquad (1).$$

Again for the star S situated at about the same distance south of Z or having about the same altitude A′ and refracton r', we have

$$\text{lat.} = 90 - (\epsilon + A' - r' - d') \qquad (2).$$

adding equations 1 and 2, we have

$$2\,\text{lat.} = A + \epsilon - r - 90 + d + 90 - \epsilon - A' + r' + d'$$

$$= A - r + d - A' + r' + d'$$

but as $A = A'$ nearly, we have $r = r'$, and hence

$$2 \text{ lat.} = A - A' + d + d'$$

$$\text{lat.} = \frac{A - A' + d + d'}{2}$$

Hence we see by this method that the errors of refraction and index error are eliminated. The importance of getting rid of these errors cannot be over estimated, and hence to derive the greatest advantage from the method, the atmosphere should be in the same condition during both observations; and, therefore, the observations should be taken as nearly at the same time as possible.

The following stars will be found convenient for this observation :—

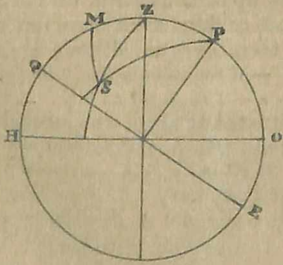
		Dec., N.	Alt. in this Latitude.
	α Ophiuchi	12° 39′	51° 15′
Regulus	α Leonis	12° 36′	51° 12′
	α Herculis	14° 32′	53° 8′
Algenib	γ Pegasi	14° 27′	53° 3′
Markak	α Pegasi	14° 29′	53° 5′
Altair	α Aquilæ	8° 31′	47° 7′
Aldebaran	α Tauri	16° 14′	54° 50′
	β Leonis	15° 18′	53° 54′
Arcturus	α Böotis	19° 52′	58° 28′
	ε Pegasi	9° 16′	47° 52′
	α Serpentis	6° 50′	45° 26′
	γ Leonis	20° 30′	59° 6′
	α Orionis	7° 22′	45° 58′
Procyon	α Canis Minoris	5° 33′	44° 9′

Those of course will be selected, when practicable, which are nearest in altitude to Polaris, but good results may be expected from any of these stars.

EX-MERIDIAN OBSERVATIONS.

LATITUDE BY A SINGLE ALTITUDE OUT OF THE MERIDIAN, THE TIME BEING SHOWN.

Suppose we have observed S, the sun or a star at a known instant of time. Then in triangle P Z S we have given $P S = 90° - S T = 90° - \delta$ $Z S =$ zenith distance $= z =$ complement of true altitude, and also



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the hour angle Z P S. To find Z P the co-latitude and hence latitude $= \phi$ suppose. Draw S M perpendicular to P Z Q. Then we have by Napier's Rules

$$\begin{aligned} \text{cos. } P &= \text{tan. } PM \text{ cot. } PS \\ &= \text{tan. } PM \text{ tan. } \delta \\ \therefore \text{tan. } PM &= \text{cos. } P \text{ cot. } \delta \quad \dots \quad \dots \quad \dots \quad (1) \end{aligned}$$

Now

$$\text{Cos. } PM : \text{cos. } ZM :: \text{cos. } PS : \text{cos. } ZS$$

$$\text{but } ZM = PM - PZ = PM + \phi - 20^\circ$$

$$\therefore \text{cos. } PM : \text{sin. } (PM + \phi) :: \text{cos. } PS : \text{cos. } ZS$$

$$:: \text{sin. } \delta : \text{cos. zenith distance.}$$

$$\therefore \text{sin. } (PM + \phi) = \frac{\text{cos. zenith dis.} \times \text{cos. } PM}{\text{sin. } \delta} \quad \dots \quad \dots \quad (II)$$

P M is found from I, and II gives the value of $PM + \phi$

$\therefore$ difference gives ϕ the latitude required.

The first equation fully determines P M, which is greater or less than 90°, according as the polar distance is greater or less than 90°; if P M be greater than 90°, then $PM + \phi$ greater 90°; if P M be less than 90°, then since the approximate latitude is always known, we can determine whether $PM + \phi$ is greater or less than 90°.

It will be remembered that the intervals from the meridian *must* be taken in solar time if the sun be observed, and in sidereal time if a star; or if not convenient, these intervals must be converted into their corresponding equivalents, before being turned into angles, to get the angle P.

The latitude by an altitude of Polaris can be found by this method, at any position in its apparent orbit. The work can be more advantageously performed, however, by the method given in the *Nautical Almanack*, the proof of which follows. The above method of finding latitude is deficient in accuracy when the object observed is more than one hour from the meridian, as the error in latitude produced by an error in the time increases very rapidly as the star leaves the meridian, and it is greatest when the star is on the prime vertical; see page

TO FIND THE LATITUDE BY AN ALTITUDE OF POLARIS AT ANY TIME.

This may be done by the method explained above, or more simply by taking out the corrections from the *Nautical Almanack* Tables, pages 501-503, and applying them to the true altitude, *i.e.*, the altitude corrected for index error, and refraction. But as it is perhaps better to see how the tables are constructed, the proof is herewith attached. This observation is specially useful at sea, as from the very slow motion of Polaris any little inaccuracy in the *time* will not vitiate the result.

In the figure

let $PS = p$ the star's polar distance; this is at present a small quantity, less than 1° 30′

Let hour angle $ZPS = t$, let ϕ the lat., and h the corrected altitude; then we might obtain ϕ in the usual way from the spherical angle PZS, but ϕ can be obtained more easily as follows :—

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$$\text{Since } \cos.\, t = \cos.\, \text{Z P S} = \frac{\sin.\, h - \sin.\, \phi \cos p}{\sin.\, p \cos.\, \phi}$$

we have $\sin.\, h = \sin.\, \phi \cos.\, p + \cos.\, t \sin.\, p \cos.\, \phi$ (1)

Now p being small, we may develop p in a series of ascending powers of p, and then employ as many terms as we require to obtain any degree of accuracy. The altitude cannot differ from ϕ by more than p. Let it differ by x,

$$\text{then we have } \phi = h \pm x, \text{ and}$$

$$\sin.\, \phi = \sin\, (h - x) = \sin.\, h \cos.\, x - \cos x.\, \sin.\, h$$

$$\text{but } \cos.\, x = 1 - \frac{x^2}{2} + \frac{x^4}{2\text{-}3\text{-}4}\ \&\text{c.}$$

$$\text{and } \sin.\, x = x - \frac{x^3}{2\text{-}3} + \frac{x^5}{2\text{-}3\text{-}4\text{-}5}\ \&\text{c.}$$

we have, by substituting

$\sin.\, \phi = \sin.\, h - x \cos.\, h - \frac{1}{2} x^2 \sin.\, h + \frac{1}{6} x^3 \cos.\, h$, and similarly
$\cos.\, \phi = \cos.\, h + x \sin.\, h - \frac{1}{2} x^2 \cos.\, h - \frac{1}{6} x^3 \sin.\, h$, and
also $\sin.\, p = p - \frac{p^3}{2\text{-}3}$ &c., and $\cos.\, p = 1 - \frac{p^2}{1\text{-}2} +$ &c.

substitute these values in formula (1) and we have

$$\begin{aligned} x = {} & p \cos t. \\ & - \tfrac{1}{2} (x^2 - 2x\, p \cos. + p^2) \tan.\, h. \\ & + \tfrac{1}{6} (x^3 - 3x^2 p \cos t. + 3xp^2 - p^3 \cos t.) \qquad (a). \end{aligned}$$

For a first approximation we have $x = p \cos t.\ \therefore x^2 = p^2 \cos.^2 t$, substitute this in second term (a) and we have for a second approximation

$$x = p \cos t. - \tfrac{1}{2} p^2 \sin.^2 t \tan.\, h. \qquad (b).$$

substituting this value of x in second and term (a), we have

$$x = p \cos.\, t - \tfrac{1}{2} p^2 \sin.\, ^2 t \tan.\, h + \tfrac{1}{3} p^3 \cos.\, t \sin.^2 t$$

The maximum value of $\frac{1}{3} p^3 \cos.\, t \sin.^2 t$ is less than 5″ and can be neglected, hence

$$x - p \cos.\, t - \tfrac{1}{2} p^2 \sin.^2 t \tan.\, h$$

$$\text{and } \phi = h - p \cos.\, t + \tfrac{1}{2} p^2 \sin.^2 t \tan.\, h.$$

The hour angle t is deduced from the sidereal time of observations and the stars known right ascension by formula $t = \text{T} - a$.

As before stated the *Nautical Almanack* contains tables to facilitate the computation.

The first table gives the values of $p \cos.\, t$, the "argument" being the sidereal time of observation (if mean time be used, simply convert into sidereal time) which differs from t by a constant quantity. This first correction applied to h gives an approximate latitude $h - p \cos.\, t$.

The second table gives the value of the term

$$\tfrac{1}{2} p^2 \sin.\, 1'' \tan.\, h \sin.^2 t.$$

The arguments being the sidereal time and the approximate latitude just found.

A third table is also given to allow for the difference between the actual value of p, and that value which is employed in the first table. The "argument" is the *date* and the sidereal time.

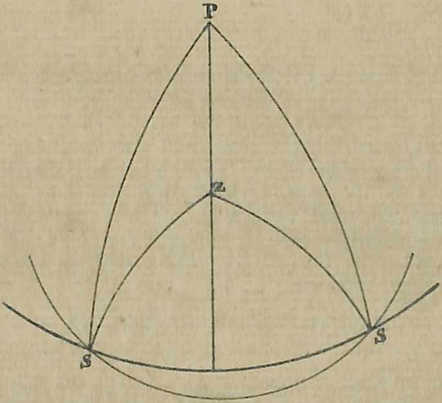
The "argument" of a table is the known quantity which serves to determine the value of some dependent function.

This proof is simply attached for satisfaction, but the tables may be practically used and taken for granted as to their accuracy.

In the above methods it has been necessary to know either the local or sidereal time of observation, but by the following methods, this knowledge is not necessary, all that is requisite is the interval between the observations.

LATITUDE BY EQUAL ALTITUDES OF ANY STAR.

Suppose we observe a star in the east, and note the time (sidereal), and again note the time when it arrives at the same altitude in the west, we can, by taking the difference of the times, get the interval, and the hour angle P from the meridian will be half this interval turned into degrees, minutes, and seconds.



$\therefore$ in the $\triangle$ Z P S
we have the angle Z P S and the side P S and the side Z S, and hence we can by the method already explained (see page) find P Z or the co-latitude.

TO FIND THE LATITUDE BY SIMULTANEOUS OBSERVATIONS OF THE ALTITUDES OF TWO KNOWN STARS.

Let S, S′ be known stars.
Z, Z′ their zenith distances, the complements of their observed altitudes.
Δ, Δ' their known polar distances.
a, or S P S′, the known difference of their right ascensions.

In the triangle S P S′, the two sides P S, P S′, and the contained angle S P S′ (the difference of their R A*s* $= a$,) are known; whence the third side S S′ and the angle P S S′ are known.

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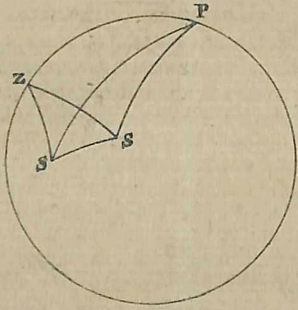


In the Δ S Z S$'$ we know the three sides and hence we can find the angle Z S S$'$; $\therefore$ the angle P S Z = (P S S$'$ − Z S S$'$) is known, and finally in the Δ Z P S we have the two sides Z S and P S and the included S and hence Z P the colatitude is known :—

In the morning and evening twilight at sea this is a very useful observation, as the horizon is usually well defined. Moreover, if the same two stars are used, the values of S S$'$ and P S S$'$ are constant, and may be tabulated for certain pairs of fixed stars. Then only Z S S$'$ and Z P remain to be determined. For conditions necessary to insure greatest accuracy, see page .

LATITUDE BY TWO OBSERVATIONS OF THE ALTITUDE OF THE SUN AND THE ELAPSED TIME.

This, it will be seen, is virtually the same as the last problem. The declination of the sun at the two times of observation replacing the declinations of the stars, and the interval of time, reduced to degrees at the rate of 360° for 24 hours, giving the angle S P S$'$, which was the difference of right ascension in the two stars.

A modification, however, arises in the case of the sun from the fact that his declination changes a little in the interval between the observations, and may be considered the same as at the *middle* time between the two observations.

The triangle (see last figure) S P S$'$ thus becomes an isosceles triangle, and, bisecting it by a perpendicular from P (not drawn in fig.), we have by Napier's rules

$$\text{Sin. } \tfrac{1}{2}\ \text{S S}' = \text{sin. } \Delta \text{ sin. } \tfrac{1}{2}\,a$$

$$\text{Cot. P S S}' = \text{cos. } \Delta \text{ tan. } \frac{a}{2}$$

The remainder of the solution will proceed as before.

The same solution will obviously apply to two altitudes of the *same star* and the elapsed time, provided this elapsed time be reckoned in sidereal tin

CIRCUM-MERIDIAN ALTITUDES.

If we take a number of altitudes of the sun or of a star on each side of the meridian, these observations are called circum-meridian altitudes when P is small; and each altitude reduced to the meridian gives nearly as accurate a result as if taken on the meridian, and if these be added together, and the mean taken, the result will be probably more accurate than that obtained from a meridian altitude, as errors of observation will neutralize each other. As hereunder shown, the value of this small reduction is $x = \frac{2 \sin.^2 \frac{1}{2} P}{\sin. 1''} \times \frac{\cos \phi \cos. \delta}{\sin. Z}$

$$= \frac{2 \sin.^2 \frac{1}{2} P}{\sin.^2 1''} \times C \text{ where } C = \frac{\cos. \phi \cos. \delta}{\cos. z}$$

Therefore if h_1 be the meridian altitude h' h'', &c., the observed altitudes, we have

$$h_1 = h' + C \times \frac{2 \sin.^2 \frac{1}{2} P}{\sin. 1''}$$

$$\text{also} \quad h_1 = h'' + C \times \frac{2 \sin.^2 \frac{1}{2} P'}{\sin. 1''}$$

$$h_1 = h''' + C \times \frac{2 \sin.^2 \frac{1}{2} P''}{\sin. 1''}$$

$$\text{and} = \text{\&c.}$$

$$\therefore h_1 = \frac{h' + h'' + h''' + \text{\&c.}}{n} + C \times \left\{ \frac{2 \sin.^2 \frac{1}{2} P + \sin.^2 \frac{1}{2} P' + 2 \sin.^2 \frac{1}{2} P'' + \text{\&c.}}{n \sin. 1''} \right.$$

in which n is the number of observations,

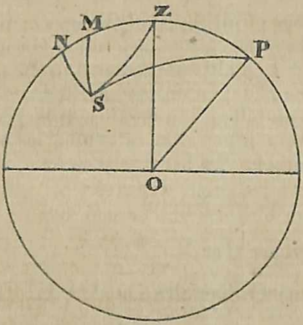
$$\text{or } h_1 = h + C \times m$$

in which h = mean of observations and m the mean of the values corresponding to $\frac{2 \sin. \frac{1}{2} P,}{\sin. 2''}$ &c., where P P, &c., are the hour angles from the meridian corresponding to h' h'', &c., and are obtained by taking the difference between the noted time of each observation, and the time of meridian passage found from the *Nautical Almanack.* When h_1 has been found the latitude is thence deduced as in the case of a meridian altitude, observing that the declination of the sun corresponding to the mean time of observation must be used.

Let P be the pole.
Z ,, zenith.
S ,, star near meridian.
M ,, point where star crosses the meridian.
PS ,, an hour circle passing through star.

Now to compute this reduction.

Suppose the altitude SM′ of the star has been taken and corrected for index error, refraction (and also for parallax and semi-diameter if the sun or a planet has been observed), it is required to compute Z M the zenith distance of the star when on the meridian. Of course we obtain the true value of Z S from the corrected altitude.



Now from the figure we perceive that

$$\left.\begin{array}{r}\text{Where}\\ \phi = \text{lat.}\\ \delta = \text{dec.}\end{array}\right\}\quad \begin{array}{l} P\,S = 90 - \delta\\ P\,Z = 90 - \phi\\ ZM = PS - PZ = \phi - \delta\end{array}$$

= meridian zenith distance of star (= Z suppose.)

With Z as a centre describe the arc S N, and the point N will be at the same altitude as S. What we want to find is MN (= x suppose) the quantity the star must increase in altitude from S before it reaches the meridian. From trig. formula we know that

$$\cos. a = \cos. b \cos. c + \sin. b \sin. c \cos. A.$$

$$\text{and } \cos. A = 1 - 2 \sin.^2 \frac{A}{2}$$

$$\therefore \cos. a = \cos. b \cos. c + \sin. b \sin. c - 2 \sin. b \sin. c \sin.^2 \frac{A}{2}$$

Also, $\cos. b \cos. c + \sin. b \sin. c = \cos. (b - c)$

$$\therefore \cos. a = \cos. (b - c) - 2 \sin. b \sin. c \sin.^2 \frac{A}{2}$$

And applying this to $\triangle$ Z P S, and calling Z P S = P, we have

$$\begin{aligned}\text{ccs. } Z\,S &= \cos. (P\,S - P\,Z) - 2 \sin. P\,Z \sin. P\,S \sin.^2 \frac{P}{2}\\ &= \cos. (P\,M - P\,Z) - \quad ,, \qquad ,,\\ &= \cos. Z - 2 \cos. \phi \cos. \delta \sin.^2 \frac{P}{2} \quad \ldots \quad \ldots \text{(I)}\end{aligned}$$

But $Z\,S = Z\,M + x$

$$\therefore \cos. Z\,S = \cos. Z\,M \cos. x - \sin. Z\,M \sin. x$$

and x being small we may say $x = \sin. x$

$$\cos. ZS = \cos. Z\,M \cos. x - \sin. Z\,M\, x$$

$$\text{and } \cos. x = 1 - \frac{x^2}{2} + \&c. \&c.$$

CSL

$$\therefore \cos. ZS = \cos. ZM\left(1 - \frac{x^2}{2} + \&c.\right) - x \sin. ZM$$
$$= \cos. Z - \tfrac{1}{2}x^2 \cos. Z - x \sin. ZM$$

∴ equation (1) becomes

$$\cos. Z - \tfrac{1}{2}x^2 \cos. Z - x \sin. Z = \cos. Z - 2 \cos. \phi \cos. \delta \sin.^2 \frac{P}{2}$$

$$\text{or } \tfrac{1}{2}\,{}^2x \cos. Z + x \sin. Z = 2 \cos. \phi \cos. \delta \sin. \frac{{}^2P}{2}$$

If we neglect the term containing x^2 (being very small), and supposing x to be expressed in seconds, we have

$$x = \frac{2 \sin. \frac{{}^2P}{2}}{\sin. 1''} \times \frac{\cos. \phi \cos. \delta}{\sin. Z} \qquad \text{(I)}$$

which formula is sufficiently accurate supposing the interval from the meridian be not greater than *ten minutes.*

The values of $\dfrac{2 \sin. \frac{{}^2P}{2}}{\sin. 1''}$ are tabulated for use, so the only part to work out is $\dfrac{\cos. \phi \cos. \delta}{\sin. Z}$ which is constant for all the observations. This reduction x must always be additive to the altitude, and consequently subtractive from the zenith distance, for an upper culmination.

In this reduction it will be well to note, that if the sun be observed, the intervals from the meridian must be taken in mean solar time, or, if in sidereal time, converted into mean solar time; and if a star be observed, the intervals must be taken by the sidereal clock, or, if by the chronometer, the intervals must be converted into sidereal time.

For a star this is all the correction to reduce the observations to the meridian; but, as the sun will have changed his declination in the interval between the observations, a further correction will be found necessary.

Let the change of sun's declination in one minute be denoted by $d\,\delta$, which is positive when the sun is approaching the elevated pole; and, if P be the sun's hour angle at the time of observation (which is negative before the sun arrives at the meridian, and afterwards positive), the whole change in declination is $P\,d\,\delta$, which is the correction to be applied to the altitude (corrected for the star quantity).

$$\left.x = \frac{2 \sin.^2 \frac{P}{2}}{\sin. 1''} \; \frac{\cos. \text{dec.} \cos. \text{lat.}}{\sin. ZD}\right\} \text{ to obtain the true meridian altitude.}$$

As we take a considerable number of altitudes (say, 10 before and 10 after the meridian passage), the mean of the values of $P\,d\,\delta$ must be applied as the correction to the mean of the altitudes.

Now, the hour angles having opposite signs on the different sides of the meridian, if we make E = sum of the hour angles observed on the east side of the meridian, and W = the sum of the angles on the west $(E - W)\,d\,\delta$, will be the correction for the sum of the observed distances; and if n be the number of observations, the mean correction to be applied to the mean of all the observed zenith distances will be $\frac{d\delta}{n}$ (E − W) where E and W are expressed in *minutes of time*, *i.e.*, are the sum of intervals from the meridian time of transit, at which the various observations were made.

It may be convenient to remember that for this correction, if the sun *be approaching the elevated pole*, and the sum of the E angles (intervals in minutes) be larger than the sum of W angles ADD the correction to the *altitude*.

If the sum of the west angles be larger SUBTRACT. Again, if the sun *be leaving the elevated pole*, and the sum of the east angles be larger than the sum of the west, SUBTRACT the correction from the *altitude*. And *vice versa* if the sum of the west be larger.

Of course just the opposite will be the case if we apply the correction to the zenith distances.

There sometimes appears to be some difficulty in knowing how to correct for change of declination at any time or at any place; and, as we are just going on to consider the question of obtaining latitude by observations off the meridian, a few words will get rid of the apparent uncertainty as to how to proceed.

Suppose we want to find the sun's declination at 9 a.m. on any date (say, Feb. 25, 1870) in longitude east 2 min. 8·802 secs.

The astronomical time corresponding to Feb. 25 at 9 a.m. is, Feb. 24, 21 hours, and the longitude, being east, must be subtracted from 21 hours to give us the corresponding *Greenwich* astronomical time—viz., 21 hrs. − 2 min. 8·802, or 20 hrs. 57 min. 51·2 secs., for which we want to find the declination.

Now, in the *Nautical Almanack*, page 2, we find that the whole change in declination, between Feb. 24th mean noon and Feb. 25th mean noon, is 22′ 14·9″, and we have

24 hrs. : 20 hrs. 57 min. 51·2 secs. :: 22′ 14·9″
from which we get x = 19′ 25·2″

to be subtracted from the declination at mean noon on the 24th to give us the declination at 9 a.m. on the 25th.

Now, dec. at Greenwich mean noon on 24th ... ... = 9° 25′ 17·3″
and, subtracting x ... ... = 0° 19′ 25·2″
we get dec. at 9 a.m., 25th ... ... = 9° 5′ 52·1″

In making these corrections to the given declinations, it is only necessary to be careful, as to whether the declination is *increasing* or *decreasing*.

To find the sun's R A at any place or any date proceed just in a similar way as for its declination, only right ascensions will of course always be *increasing*.

If it be required to find the sun's declination at *apparent* noon at any place (say for instance Chatham), or at a place 2 min. 8·8 secs. E of Greenwich (for example) on the 25th Feb., 1870.

The apparent astronomical time at Greenwich on 24th Feb. is 23 hrs.

GSL

57 min. 51·2 secs. The "difference for an hour" is the difference *at noon*, and on 24th Feb. is 55·44, and on 25th Feb. is 55·79″ ∴ for midway between Feb. 25th at noon and Feb. 25th 23 hrs. 57 min. 51·2 secs. it is 55·7895, which multiplied by 2 min. 8·802 secs. (= $\frac{1}{30}$ of an hour nearly) gives 1·789475″ to be added to declination at Greenwich apparent noon on 25th.

So that, declination at Chatham at apparent noon on 25th Feb., 1870, was

$$\begin{array}{r} 9^\circ\ 2'\ 50{\cdot}1'' \\ +\quad 1{\cdot}789475'' \\ \hline 9^\circ\ 2'\ 51{\cdot}889475'' \end{array}$$

It will be noticed that the approximate result may be obtained at once by using the "difference in an hour" as given in *Nautical Almanack.*

The method just given supposes P to be known with tolerable accuracy, when this is not the case we may proceed to obtain the true value of P as follows:—

Let Z' and Z'' be two observed zenith distances.

b and b' the corrections to reduce them to the meridian.

h and h' the corresponding hour angles less the $10'$

from formula (1) we have

$$b = \frac{2\sin.^2 \frac{1}{2} h^2}{\sin 1''} \times \frac{\cos. d \cos. \phi}{\sin. Z} = \frac{225\, h^2 \sin. 1''}{2} \times \frac{\cos. \delta \cos. \phi}{\sin. Z} = c h^2$$

similarly

$$b' = c\, h'^2$$

$$\text{and therefore } b - b' = c\,(h^2 - h'^2)$$
$$= c\,(h + h')\,(h - h')$$

$$\text{but } Z' = Z + b \text{ where } Z = \text{meridian zenith distance.}$$
$$Z'' = Z + b' \text{ hence } Z' - Z'' = b - b'$$
$$\therefore Z' - Z'' = c\,(h + h')\,(h - h').$$

Now $h + h'$ or $h - h'$ is known, being the interval of time between the two observations according as they are taken on the opposite or the same side of the meridian. For argument sake suppose them to be taken on the same side, then $h - h' = t$ is known, and hence

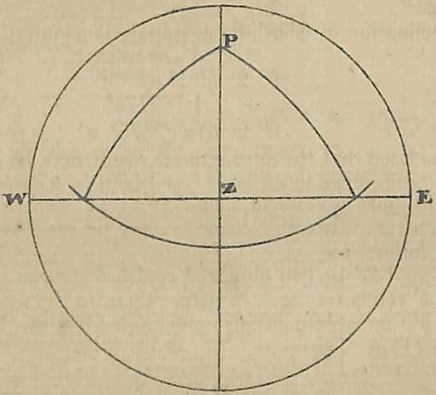
$$h + h' = \frac{Z' - Z''}{c\,(h - h')} \text{ is also known}$$

and hence h and h'

The above method of finding latitude is susceptible of its greatest accuracy when the observations are taken at equal intervals on opposite sides of the meridian.

LATITUDE BY TRANSITS ACROSS PRIME VERTICAL.

This method supposes the transit instrument to be placed with its supports North and South, so that the axis of the transit points due East or West. We must then from the noted times of passage of the known star across the different wires, deduce the time of passage over the mean wire, having done this, for the Eastern and Western transits, we know the interval in time between the two, or the hour angle, and hence we may find the latitude as follows :—



Let P represent the pole of the earth, W E the prime vertical, Z being the zenith, and S S′ the circle of declination described by the star. Then if we know the declination (d) of the star, and the angle S P S′ we know the polar distance P S ($= 90 - d$) and the angle $ZPS = \frac{1}{2}$ S P S′, and therefore by Napier's rules we can find in the spherical right angled triangle the side P Z which is the colatitude as follows :—

Taking Z P S as the middle part or

$$\sin. (90 - ZPS) = \tan. ZP \tan. (90 - PS)$$

$$\text{or } \cos. ZPS = \tan ZP \tan. d$$

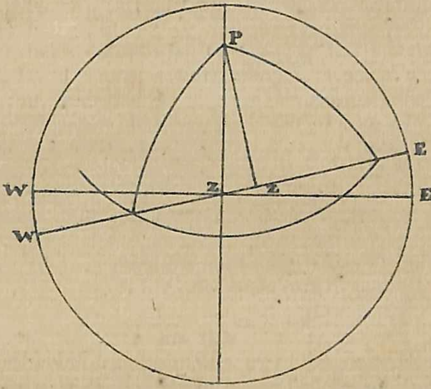
$$\cos. \frac{P}{2} = \cos. \text{lat.} \times \tan. d$$

$$\text{hence} \quad \cos. \text{lat.} = \frac{\cos. \frac{1}{2} P}{\tan. d}$$

The value of the latitude by this method is unaffected by a constant error of the clock; and the best result is obtained when the star for observation has a declination about equal to the latitude of the place of observation. From the above equation it will be observed that if the latitude of the place of observation be known the declination of the star can be found.

The transit instrument can be placed nearly in the prime vertical, by calculating the angle P, by the above equation, and moving the instrument so that the star is on the middle wire at the moment that the clock shows the calculated time—or by taking the mean of the East and West transits, and comparing them with known R. A. of star, the difference shows angle the transit makes with prime vertical.

And this correction of the deviation (Z P Z′) may be calculated as follows :—



Now from the two right angled triangles Z P Z′ and Z P S we have as follows, taking Z P Z′ and Z′ P S′ as the middle parts.

$$\sin.\ (90 - \mathrm{Z\,P\,Z'}) = \tan.\ \mathrm{P\,Z'}\ \cot.\ \mathrm{P\,Z}$$

$$\cos.\ \mathrm{Z\,P\,Z'} = \tan.\ \mathrm{P\,Z'}\ \cot.\ \mathrm{P\,Z}$$

Similarly

$$\cos.\ \mathrm{Z\,P\,S} = \tan.\ \mathrm{P\,Z'}\ \cot.\ \mathrm{P\,S}$$

$$\frac{\cos.\ \mathrm{Z\,P\,Z'}}{\cos.\ \mathrm{Z\,P\,S}} = \frac{\cot.\ \mathrm{P\,Z}}{\cot.\ \mathrm{P\,S}} = \frac{\tan.\ \mathrm{lat.}}{\tan.\ \mathrm{decl.}}$$

$$\therefore \quad \cos.\ \mathrm{Z\,P\,Z'} = \cos.\ \tfrac{1}{2}\,\mathrm{P}\,\frac{\tan.\ \mathrm{lat.}}{\tan.\ \mathrm{decl.}}$$

For further particulars as to this method see Loomis, Chauvenet, etc.

EFFECTS OF ERRORS OF DATA ON LATITUDE BY A SINGLE ALTITUDE.

$$\text{From the formula } \cos.\ \mathrm{P} = \frac{\cos.\ a - \cos.\ b\ \cos.\ c}{\sin.\ b\ \sin.\ c}$$

we have,

$$\left.\begin{array}{l} a = 90 - h \\ b = 90 - d \\ c = 90 - \phi \end{array}\right\} \text{ where } h,\ d, \text{ and } \phi \text{ is the altitude, declination, and latitude.}$$

and, therefore, by substitution

$$\cos.\ \mathrm{P} = \frac{\sin.\ h - \sin.\ d\ \sin.\ \phi}{\cos.\ d\ \cos.\ \phi} \text{ or}$$

$$\sin.\ h = \sin.\ d\ \sin.\ \phi + \cos.\ \mathrm{P}\ \cos.\ d\ \cos.\ \phi$$

Now, if we differentiate this equation on the supposition that h, ϕ, and P are variables, and replacing P by its value in terms of the azimuth A, we have

$$d\,h = -\cos.\ \mathrm{A}\ d\,\phi - \cos.\ \phi\ \sin.\ \mathrm{A}\ d\,\mathrm{P} \qquad (1')$$

$$\text{or, } d\,\phi = d\,h\ \sec.\ \mathrm{A} - \cos.\ \phi\ \tan.\ \mathrm{A}\ d\,\mathrm{P}$$

from which it appears, $d\,\phi$ is a minimum, when cos. A is a maximum and sin. A a minimum, *i.e.*, when $A=o$, or 180—or when the body is on the meridian.

$d\phi$ is a maximum when cos. A is a minimum or when $A=90°$, and in this case the error in ϕ corresponding to errors in P and h is a maximum when the body is on the prime vertical. This is the worst time to take an observation. If we suppose the time to be accurately known, then $dP=o$, and we have $d\phi=dh \sec. A$; a maximum when $A=90°$, a minimum when $A=O$.

If we suppose $dh=o$, then
$d\phi=\cos.\ \phi \tan.\ A\ dP$, a maximum when $A=90°$, a minimum when $A=o$.

Also, in equation (1), if $d\phi=o$ then we have

$$dh=\cos.\ \phi \sin.\ A\ dP$$

$$\cos.\ \phi=\frac{dh}{dP}\ \frac{1}{\sin.\ A}$$

Suppose, now, that we take two altitudes, at a few minutes interval, of the sun or other heavenly body, and we note the time corresponding to the small increase of change, then dh may be considered the change of altitudes due to dP the noted interval, and if the body is on the prime vertical, or within a few minutes of it, $\sin.\ A=1$, and we have

$\cos.\ \phi=\frac{dh}{dP}$, and hence an expeditious method of finding an approximate latitude when sun is on the prime vertical; and if on prime vertical when rising or setting, by observing the time the sun's diameter takes to rise above or sink below the horizon, dividing the sun's diameter by this time we obtain an approximate latitude to within a few minutes. No knowledge of time or declination required by this method.

SIMULTANEOUS OBSERVATIONS OF THE ALTITUDES OF TWO KNOWN STARS OR SAME STAR, WITH TIME BETWEEN, OR EQUAL ALTITUDES.

Since the hour angles are connected by the equation $P=P'+\lambda$ where λ is the angle between the two stars; we have $dP=dP'$

Also, we have the general relation

$$\sin.\ h=\sin.\ \phi \sin.\ d+\cos.\ P \cos.\ d \cos\ \phi$$

$$\sin.\ h'=\sin.\ \phi \sin.\ \delta'+\cos.\ P' \cos.\ d' \cos.\ \phi$$

which, by differentiation relatively to h, ϕ and P, and substituting for P and P' their values in terms of A and A' the azimuths, we have

$$dh=-\cos.\ A\ d\phi-\cos.\ \phi \sin.\ A\ dP$$

$$dh'=-\cos.\ A\ d\phi-\cos.\ \phi \sin.\ A'\ dP$$

A and A' being the azimuths of the two stars, or the azimuths of the same star at each observation.

Solving for dP and $d\phi$ we find

$$d\phi=-\frac{\sin.\ A'}{\sin.\ (A'-A)}dh+\frac{\sin.\ A}{\sin.\ (A'-A)}dh'$$

$$\cos.\ \phi\ dP=\frac{\cos.\ A'}{\sin.\ (A'-A)}dh-\frac{\cos.\ A}{\sin.\ (A'-A)}dh'$$

These equations show that, in order to reduce the effects of errors of altitudes as much as possible, we must make $\sin.\ (A'-A)$ as great as possible, and hence we get when $\sin.(A' - A) = 1$, or $A' - A = 90^\circ$.

$$d\phi = -\sin. A' \, dh + \sin. A \, dh'$$

$$\text{and } \cos.\phi \, dP = \cos. A' \, dh - \cos. A \, dh'$$

and, under this supposition, if one of the altitudes is taken on the meridian the other should be taken on the prime vertical; if then the altitude h is on the meridian sin. A will be small, while sin. A′ nearly equal unity, and hence, $d\phi = -dh$; at the same time cos. A′ will be o, and $\cos. A = 1$ and $\cos.\phi \, dP = -dh$, and hence the accuracy of the latitude will depend on the altitude near the meridian, and the time on the altitude near the prime vertical; if the observations are taken on the same star at equal distances from the meridian, we have $A = A'$ and

$$\text{therefore } d\phi = \frac{2\cos. A}{dh + dh'}$$

$$\cos.\phi \, dP = -\frac{2\sin. A}{dh - dh'}$$

and hence the most favorable condition for determining latitude and time from equal altitudes is when $\sin. A = \cos. A$, or $A = 45^\circ$

LONGITUDE.

The interval between the two successive transits of a star over the same meridian is always equal to 23 hours 56 minutes 4·091 seconds mean solar time.

The interval between two successive transits of the sun is always *more* than this, and the *average* is 24 hours solar time—*i.e.*, 24 hours for 360° or 1 hour for 15°.

It is evident that the sun in his daily apparent path will cross the meridian of different places, and that noon will occur later at some places than at others. That is, in as far as they are west of some fixed standard meridian.

Suppose Greenwich to be the standard

Then the sun will cross the meridian of a place, 15° west of Greenwich, one hour later than it crossed Greenwich meridian—in other words, *at one o'clock Greenwich time*, and so on for other places, at the rate of 15° an hour, or 360° in twenty-four hours. Such being the case, if, at any part of the earth's surface, we can ascertain our *local* mean time, and compare it, by any means, with Greenwich mean time, and then convert the difference of time into degrees, minutes, and seconds, at the rate of 15° an hour, we have found our longitude.

There are four methods of determining the longitude which may be defined as the (1st), astronomical, (2nd), chronometric, (3rd), telegraphic, and (4th), the geodetical methods; and let us, first, consider the astronomical methods depending on the moon's motion among the stars, in which she may be likened to the hands of a clock, the dial being the sky, and the fixed stars points to measure from. Owing to the revolution of the moon round the earth in the period of about 28 days, her place changes among the stars more rapidly than that of any other heavenly body. Her place changes roughly speaking at about the rate of half a degree per hour, or about her own diameter.

GSL

Suppose then that at Greenwich on a certain day when the moon is on the meridian, a certain star is seen to be in exact contact with her edge, and let us suppose that the revolution of the earth brings the moon to the meridian of a spot, whose longitude is required one hour after the Greenwich transit. The star which was seen in contact with the moon's edge, will in consequence of the progressive motion of the moon, be now separated by, roughly speaking half a degree, or as above by a space about equal to the moon's diameter. Hence it is easy to see that by measuring the relative distances between a star and the moon at any two places on the earth's surface we shall be able to ascertain the difference of longitude, provided we know the rate at which the moon was advancing in her orbit round the earth.

Thus making use of the position of the moon to determine the longitude we have the following means of doing so :—

1st.—By R. A. of the moon.

2nd.—Observation of its true position among stars at instants of time accurately noted.

3rd.—Occultations of stars by the moon and eclipses of Jupiter's satellites, &c.

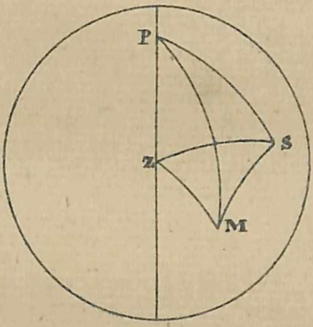
4th.—Eclipses of the sun.

By the first method we can find out the longitude by determination of moon's R.A. when on or off the meridian, by the following methods:—

1st.—If we note the time of transit of the moon's bright limb over each wire of the transit instrument, we can find out the time of transit over the mean wire, and knowing the allowances to be made for errors of level, collimation and azimuth, we can find the instant of the transit of the moon's bright limb over the meridian; also knowing the value of the moon's semi-diameter, we have then the R.A. of the moon's centre at the moment of passing our meridian. From the *Nautical Almanack* we can find by interpolation the Greenwich time corresponding to this R.A., the difference between the two times is the longitude required.

The moon's right ascension may also be determined by means of sextant observation as follows; observe the altitude of the moon at any time (best when moon is on prime vertical); then knowing the approximate longitude, we can find the moon's polar distance, and also the moon's semi-diameter and parallax, hence in the astronomical triangle P Z M we have the three sides, and hence we can find the hour angle Z P M, and knowing the local sidereal time of observation ($= T$); if the moon be east of the meridian, we have then its R.A. $= T + t$. By interpolation we can find the Greenwich sidereal time corresponding to this R.A., and the difference between this and noted local sidereal time, is the longitude required. The R.A. can also be determined otherwise as follows :—

Let M be the moon and S a known star; then if the altitude of the moon and star be noted, as well as distance M S, all corresponding to the same instant of time; in the $\triangle$ Z M S we can find the angle M Z S; also from the triangle P Z S, we can find the angle P Z S and Z P S $=$ (T), for we know the three sides; hence in the triangle P Z M, the two sides, and the included angle Z = (P Z S + M Z S) are known, and



hence we can determine the angle Z P M = (t), hence the angle M P S = $(T - t)$, and as the right ascension of the star (= T) is known, the right ascension of the moon becomes known, and hence as before the longitude is determined from the data supplied by the *Nautical Almanack.*

The second method of determining the longitude is subdivided into two; these are (1st) the method of moon culminating stars, (2nd) lunar distances.

The process employed in the method of moon culminating stars may be stated as follows:—

The *Nautical Almanack* contains for every day in the year a set of stars, generally four in number which are selected as being near the moon on the respective days, and so passing the meridian nearly at the same time, and altitude as the moon. The *Nautical Almanack* also gives the interval in time elapsing at Greenwich between the passage over the meridian there of each of these stars, and the bright or well defined edge of the moon. The observer at the distant station, the longitude of which is required, observes with his transit instrument adjusted to the meridian, the moments at which the stars provided for the day, and the bright edge of the moon pass the wires of the transit; these observations are reduced to the mean wire, and corrected as usual for level, collmination, and azimuth, (as the stars selected and the moon have about the same declination, and pass the meridian within a small interval of one another, the instrumental corrections affect the stars and the moon almost equally, and in the same way, and hence the intervals between the transits are unaffected by the instrumental errors. It is evident that the interval is also unaffected by a constant clock error; the difference between these intervals are

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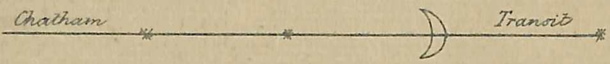
then taken and compared with the Greenwich interval, and the longitude deduced as follows :—



Suppose the above to be the position relatively of the moon and stars at the Chatham transit. By the time the earth's revolution has brought them successively on to the meridian of Greenwich, the moon from its own motion eastward among the stars, will have altered its position relatively as shown in the diagram. Now this alteration of position is called the moon's variation in right ascension, and is given for each day in the *Nautical Almanack* for one hour of longitude; so, if we take the differences of the intervals, and the mean of these differences, and compare this with "Variation of Moon's R.A. in one hour of longitude," we evidently shall obtain the longitude; thus,

Variation of R.A. in one hour of longitude : mean of difference :: 1 hour : *b*; where *b* will be the longitude in time.

This observation will give very close results, even neglecting the more delicate corrections due to the inconstancy of the moon's rate, when the difference of the meridians is not very great.

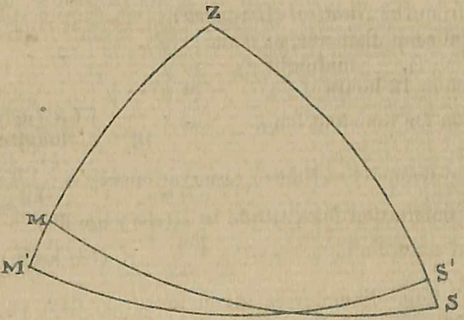
But where this difference is considerable (a couple of hours or more) and extreme accuracy is wanted; that variation in R.A. should be used, which corresponds to the middle of the interval between the observations. This value of the variation in R.A. may be obtained by interpolation for the middle interval, from the values given in the *Nautical Almanack* for successive lunar transits.

The chief advantage of this method, consists in the fact that it is not affected by constant errors of instrument, and that observations can be frequently made. Many circumstances, however, tend to produce error in the longitude so obtained; among them may be mentioned irradiation of the moon's limb which not only disturbs the observation, but casts uncertainty on the diameter of the moon; its want of sharp definition, and the still unaccountable errors in the moon's motion. The longitude may be obtained by this method to within 3 seconds when the mean of 30 or 40 observations is taken.

LUNAR DISTANCES.

The principle of this method is as follows:—

The altitude of the moon's upper or lower limb, the altitude of a known star, and the distance of the moon's rounded edge from the star, being all reduced to same local time, the observations are corrected for index error and semi-diameter only, so that in the spherical triangle Z M S, by solving, we get the accurate value of Z, then we correct the altitudes for refraction and parallax, so that the *true* altitudes and the angle Z being known, we can obtain the true distance of the moon from the star at the local time of observation; then from the N A we obtain by interpolation, the Greenwich time, corresponding to the same distance, and hence the difference between the two times gives the longitude. The method of performing the calculation is as hereunder:—To find the altitudes of the sun or star, and lunar distances at the same absolute instant of time, three observers are necessary. If only one observer is available, then the observations are performed in the following manner. The observer notes—



1st.	The mean of	several altitudes of the sun or star	= S	mean time =	t_1
2nd.	,,	several ,, moon	M	,,	t_2
3rd.	,,	several lunar distances	D	,,	T
4th.	,,	several altitudes of moon	M_1	,,	T^1
5th.	,,	several altitudes of the sun or star	S_1	,,	T_2

From these data the altitudes of the sun or star, and moon are reduced to the mean time T, corresponding to the mean lunar distance.

Now, understanding the general question, let us consider the corrections which will be applied throughout at the different stages of the calculation.

1st.—We will merely correct our observed altitudes for semi-diameter and index error, and our observed distance for semi-diameter, and index error; and then solve for z by the formulæ :—

$$\sin\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c.}} \left\{ \text{where } s = \frac{a+b+c}{2} \right.$$

$$\text{or } \cos.\frac{A}{2} = \sqrt{\frac{\sin s \sin(s-a)}{\sin b \sin c.}}$$

2nd. — We next must correct our altitudes for refraction and parallax, and then with these corrected values and the angle Z, determine M′ S′ the true lunar distance—for instance,

observed altitude = ⎧ this being done
semi-diameter = ⎩ in the first case.
index error =
apparent altitude =
refraction =
parallax =
corrected altitudes =
zenith distance =

Now this first correction for semi-diameter is obtained in the following manner from the *Nautical Almanack* :—

horizontal semi-diameter, at noon $= y'$
" " midnight $= y$
difference in 12 hours $= y' - y$
proportion for time and long. $= \dfrac{y'-y}{12}$ $\Big\{$ (× $t \pm$ approximate longitude in time.)

horizontal semi-diameter at time of observations $= y' + \dfrac{y'-y}{12} \times (t \pm \text{long.})$

augmentation for altitude $= \epsilon$ (see page 30)

corrected semi-diameter $= y' + \dfrac{y'-y}{12} \times (t \pm \text{long.}) + \epsilon$

The moon's semi-diameter is given for every day, page 3 of the *Nautical Almanack*, and the augmentation due to the altitude can always be found from tables, or by the formula given under the head of semi-diameter.

We now come to the consideration of the correction for parallax, and it will be as under for any date :—

Horizontal parallax, at noon $=$ P
" " midnight $=$ P′
difference in 12 hours $= P - P'$
correction for time and longitude $= \dfrac{P-P'}{12} \times$ $\Big\{$ $t \pm$ approximate longitude in time.)
correction for latitude $= z$

$$\therefore \text{ horizontal parallax } = \frac{P-P'}{12} \times \left\{ t \pm \text{long.} \right\} - z.$$

And then we get the parallax in altitude in the usual manner from formula.

Log. sin. parallax in altitude = log. sin. horizontal
parallax + log. cos. altitude.

In observations of the sun or planets the mean radius of the earth may be taken as the value of C O in previous figure, so that the sin. of the parallax in altitude will be sin. horizontal parallax × cos. alt. ; the moon, however, being so close to the earth, the difference in the earth's radius at different latitudes must be considered. In the *Nautical Almanack*, the moon's horizontal parallax at the equator is given for the moments of Greenwich noon and midnight throughout the year, and the horizontal parallax for any other latitude will bear the same proportion to the equatorial horizontal parallax that the earth's radius in the given latitude does the equatorial radius.

Tables are given for this purpose in most practical works on astronomy.

Having now corrected everything, we have the true zenith distances Z M′, $z\ s'$, and the angle Z, we must therefore solve the triangle Z M′ S′ in order to find M′ S′ the true distance, this can be done by means of Napier's analogies, but the following method is more simple.

The two sides and the included angle being given (a, b, and C say) ; have

$$\cos. c = \cos. b \cos. a + \sin. a \sin. b \cos. C.$$
$$= \cos. b\ (\cos. a + \sin. a \tan. b \cos. C).$$

and assuming $\tan. \theta = \tan. b \cos. C$

$$\cos. c = \cos. b\ (\cos. a + \sin. a \tan. \theta)$$
$$\text{or } \tan. \theta = \tan. Z\,M' \times \cos. Z \quad . \quad . \quad . \quad \text{I}$$
$$\cos. c = \frac{\cos. b \cos. (a - \theta)}{\cos. \theta} \quad . \quad . \quad . \quad . \quad \text{II}$$

From (I) we get θ . · . substite in (II) for $(a - \theta)$

Let $(a - \theta) = \phi$ suppose

$$\text{Then } \cos. c = \frac{\cos. b \cos. \phi}{\cos. \theta}$$
$$\text{Cos. } M'S' = \cos. \frac{ZM' \cos. \phi}{\cos. \theta}$$

where M′S′ is the correct lunar distance.

Now from the *Nautical Almanack*, pages XIII. to XVIII. we can find for every third hour of Greenwich mean time, the angular distances between the centre of the moon and certain heavenly bodies, whether sun, planet or star be observed, and also the difference of distance answering to this interval of three hours ; and therefore we can tell between which times the actual distance occurred, and taking the nearest, we have

$$\text{Lunar distance at 9 (say) Greenwich} = L$$
$$\text{,, \quad ,, as found by observation and solution as above} = y$$
$$\text{Difference} = L - y$$

Then difference in 3 hours being given (say) Z, we have

$Z : L - y : : 3$; the interval past nine.

Add 9 (say) to this, and we have the Greenwich time corresponding to the distance. We know the local time corresponding to same distance; therefore the difference gives us our longitude as before.

Instead of obtaining the interval past 9 by proportion, we might have taken it from the table of proportional logs.

These tables of logs. are constructed on the following principle :—

The proportional log. of an angle expressed in hours or degrees is the log. of the quotient of 3 hours or 3° divided by the angle. Thus if A is the angle; proportional log. $A = \log. \frac{3^h}{A}$ or $\log. A = \log. 3^h - \log. A$. The angle is always supposed to be reduced to seconds, so that in either case we have $P.\ L.\ A = \log. 10800 - \log. A$. Tables of such logs. are given in Chambers' logarithms.

If now we wish to interpolate a value of a lunar distance for a time, $T - t$, which falls between the times of the Ephemeris $T + T + 3^h$, we are to compute the correction for the interval t, and apply it to the distance given for the time T, and if we put Δ for the difference of the distance in the Ephemeris for 3 hours, and D the required correction we have by simple interpolation.

$$D : \Delta = t : (10800 \text{ seconds})$$

$$\therefore D = \frac{t \times \Delta}{10800} \text{ and}$$

$\log. D = \log. t + (\log. \Delta - \log. 10800)$. Subtract both members of this equation from log. 10800, and we have

$$\log. 10800 - \log. D = \log. 10800 - \log. t - (\log. \Delta - \log. 10800).$$

$$P\,L\,D = P\,L\,t + P\,L\,\Delta.$$

which is computed by the tables above mentioned or knowing D, t may be computed from same formula; but the first differences of the lunar distances cannot be assumed as constant, when the interval is as great as three hours; therefore, we must take into account the second differences, and we know from the formula of interpolation proved, (see p. 76) that

$$d = a + \frac{\text{sum 1st diff.}}{6}\,t + \frac{\text{sum 2nd diff.}}{18} \times t^2 \qquad (a)$$

where the distances at intervals of three hours are given.

Example.—Find the distance of the moon's centre from the star α Aquila at 4.45 Greenwich time, May 25, 1874.

				1st diff.	
38	28	24	distance at noon		
				44·6	+ 3′ 6″
39	12	30	distance at 3 p.m.		
				47·12	
39	59	42	distance at 6 p.m.		

substitute in the above formula, and we have

$$d = 39\cdot\ 12\ 30\ + \frac{91'\cdot 18}{6} \times 1\cdot\tfrac{3}{4} + \frac{3\cdot' 6''}{18} \times (1\cdot\tfrac{3}{4})^2$$

$$= 39\ 12\ 30\ +\ 26'\ 37\tfrac{3}{4}''\ +\ 31\tfrac{1}{4}''$$
$$= 39^\circ\,39'\,39'' \text{ the distance at } 4{\cdot}45 \text{ p.m.}$$

If we had neglected in this case to take into account the second differences, and supposed the moon's motion to be uniform, we would find

$$d = 39{\cdot}\ 39{\cdot}\ 2 \text{ or an error of } 37''.$$

Suppose now that having cleared a lunar distance, and we want to know the Greenwich time corresponding to this distance, it is clear that by solving the quadratic equation (a) for t, we would find t accurately; but this may otherwise be done with sufficient accuracy as follows:—As the third term of the equation (a) is small; for a first approximation we may neglect it, and hence solving the equation we have

$$d = a + \frac{\text{sum. 1st diff.}}{6} \times t \text{ where}$$

$$\begin{aligned} \text{where } d &= 39\quad 39\quad 39 \\ a &= 39\quad 12\quad 30 \\ \text{and sum 1st diff.} &= \qquad 91'\quad 18 \end{aligned}$$

we get $t = 1{\cdot}784$ a first approximation to the time; substitute this approximate value of t in the term $\frac{\text{sum 2nd diff.}}{18} \times t^2$, and we get for the value of this term $32{\cdot}8''$; again solve equation (a) for t with this value for the 3rd term, and we get a second approximation for $t = 1{\cdot}75^{\text{h}} = 1^{\text{h}}\,45^{\text{m}}$; had we neglected to take into account the second difference we would have had an error of $1{\cdot}784 - 1{\cdot}75 = 034{\cdot} = 2^{\text{m}}\,2''$ in the longitude.

This second difference may be calculated more expeditiously in the following way, by means of tables given in *Nautical Almanack*. We have seen that

$$\begin{aligned} \text{Prop. log. } x &= \text{prop. log. } t + \text{prop. log. } \Delta \\ \text{Prop. log. } t &= \text{prop. log. } x - \text{prop. log. } \Delta. \\ \text{Now } x &= 39{\cdot}\ 39{\cdot}\ 39{\cdot} - 39.\ 12{\cdot}\ 30 = 0{\cdot}\ 27'\,9'' \\ \text{and } \Delta &= \qquad\qquad\qquad\qquad\quad 0{\cdot}\ 47'\,6'' \end{aligned}$$

and taking out the proportional logs. from Chambers' tables, we have

$$\begin{aligned} \text{prop. log. } t &= 82150 - 58133 = 24017 \\ \text{hence } t &= 1^{\text{h}}\ 43^{\text{m}}\ 33''. \end{aligned}$$

Now the proportional log. standing opposite the nearest distance in the Ephemeris is 5813, those immediately to the left and right are respectively 6108, and 5553, the differences are, therefore, 395 and 240, and the mean is 278, which in this case exceeds the limit of the table which is 138; dividing therefore by 3, to bring it within the limit of the table, we have $\frac{278}{3} = 93$, and opposite to 1· 43 33 (or quantity nearerst to it 1·40) and under 93 we have the correction 28·5, which multiplied by 3 = 85·5, and this added to 1· 43· 33 as the proportional logs. are decreasing gives as the correct interval 1^{h} 44· 58·5, which is almost the same as that found above; and hence the corresponding Greenwich time is $4^{\text{h}}{\cdot}$ 44· 58·5, at which the distance

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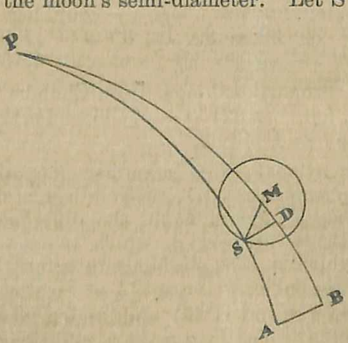
is 39° 39′ 39. The proportional logs. given in the *Nautical Almanack* serve also to point out the star best suited for observation. It is that one which has the least P.L. opposite it; for the greater the velocity of the moon to or from a star the greater is the reliance to be placed on an observation of the distance, and as it is a property of P.L. to decrease as the natural numbers increase, a smaller proportional log. indicates a greater velocity of the moon, or a greater variation of distance in the interval upon which the value of the observation depends.

Occultation of Stars and Jupiter's Satellites.—On account of the moon's large parallax, it often happens that the same star is occulted at different times to different observers, and hence we take advantage of this circumstance to find the longitude. From the local observed time of occultation we can find by calculation the true time of conjunction of the moon and star, *i.e.*, the time as seen from the centre of the earth; this phenomena happens at the same absolute instant for every observer; so that we can find the local time, corresponding to the same absolute instant of time. In the *Nautical Almanack*, the Greenwich time of conjunction is given for certain stars, so that if we calculate the local time of conjunction, and compare this with the Greenwich time, the difference between the two gives the longitude. In order to determine the local time of true conjunction we must determine from the observed instant of immersion or emersion.

1—The apparent difference of right ascension between moon and star;
2—The true difference of right ascension;
3—The true time of conjunction;

Let P represent the pole, M the centre of the moon, and S the star at the instant of immersion, its apparent distance from moon's centre is then equal to the moon's semi-diameter. Let S D be a parallel of declination passing through S, the star, and let A B be the arc of the equator, intercepted between the hour circles P A and P B; then M D is the apparent difference of declination $= d$ and $\frac{S\,D}{\cos.\ A\,S}$ their ap-



parent difference right ascension $= a$.

As the triangle S M D is small, we have $SD^2 = PM^2 - MD^2 = R^2 - d^2 = (R + d)(R - d)$

$$\text{hence } a = \frac{\sqrt{(R + d)(R - d)}}{\cos. d)}$$

Now we can calculate the moon's parallax in right ascension $= x$, and hence from the above, as a is known, the difference in right ascension between the moon's centre and the star is $a \pm x$; then to find the time of true conjunction we have only to find the time (t) the moon will take to describe the arc $a \pm x$. This can be done, once we know the moon's motion per hour for the time corresponding to middle of interval between observed time and time of true conjunction; hence the time of true conjunction $= (T \pm t)$; T being the instant of immersion or emersion.

It is clear that, if the exact moment of the immersion and emersion of one of Jupiter's satellites be noted in local time, the difference between the observed time and the calculated time, given in the *Nautical Almanack*, will be the longitude. The difficulty of applying this and last method in practice, arises from the fact, that the exact moment of immersion or emersion depends on the size of the telescope employed. Larger telescopes give later times of immersion and earlier times of emersion than smaller ones. It will be apparent that this method is useless at sea.

The first satellite passing most quickly into the shadow of Jupiter is the one best adapted for the purpose. By taking the mean time corresponding to the immersion and emersion, a good result may be obtained.

By Chronometers.—The longitude may be found, by the aid of chronometers as follows:—Before leaving the station with which the longitude of the other is to be compared, the chronometer is compared with the standard clock (the error of which is known with the greatest accuracy), and hence the chronometer error and rate are accurately known. Upon the arrival of the chronometer at the station whose relative longitude is required, it is compared with the standard clock there (showing local time), and the error of the chronometer is obtained on the local time of the second station, and as the error on the local time of the first station is known, the difference between the two gives the difference of longitude. The method just described seems very simple; but when put in practice it is one requiring great nicety and patience, for a chronometer, however perfect, is as an instrument, after all, in spite of all the skill employed upon the compensation of its balance, never an absolutely perfect time keeper. Hence many must be employed, in order that by taking the indications of all into account, the error resulting from the employment of any one may be checked. Astronomers are not, however, content with this precaution, for as Professor Forbes states in his work on the present transit of Venus, the Russians have recently introduced an absolutely uncompensated chronometer among the compensated ones, in order to measure

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the sum total of the effects of temperature to which all have been exposed. He states that "Ten well tried chronometers accompanied by a single uncompensated one, if carried between stations ten days apart (*e.g.*, St. Petersburg and Kazan) will in one journey give the longitude of an intermediate station, such as Moscow correctly within $\frac{1}{10}''$ in time." As Professor Forbes points out, in this method, the idea of adding the uncompensated chronometer is theoretically perfect if the temperature (to which errors and change of rate are mainly due) do not vary rapidly. The effect, however, of magnetism and variations of barometric pressure, on the performance of chronometers remains still uncounteracted.

Some celebrated chronometric expeditions have been made, notably by Struve between Pulkova and Altona, in 1843, in which 68 chronometers were transported nine times from Pulkova to Altona and back. That between Liverpool, and Cambridge, U.S., in 1855, by Bond, in which 52 chronometers were used, and carried backwards and forwards three times. In these long voyages not only has the ordinary rate of each chronometer, and the variation of their rate due to temperature to be determined, but also its land or sea rate, for the rate of a chronometer is rarely the same in motion as when at rest.

By Electricity—For the method of determining the longitude by the electric telegraph, we are indebted to the Officers of the United States' Coast Survey, although it was first suggested by Wheatstone in 1840. Before the Atlantic cable was laid the Officers of the United States Survey had employed the telegraph to determine the longitude of many places within their territory, from almost the extreme east to the extreme west, and they convinced themselves that the certainty of the determination was 20 to 30 times greater than that attained either by the astronomical or chronometric method. Ever since the laying down of the Atlantic cable the Coast Survey has used this, and every available cable for the purpose of determining the longitude. This method of determining the longitude is applied as follows:—The place whose longitude is known is electrically connected with that whose longitude is sought. The errors of the sidereal clocks at both places being accurately known, if a known star is observed (at the station whose longitude is known) to pass the meridian at T hours, an electrical signal is sent to the second station, and the time T' of its receipt noted. Similarly when the same star passes the meridian of the second station the time t^1 is noted, and a signal is sent to the first station where the time t of arrival is noted.

If then l be the length in miles of the electrical wire connecting the two stations, and V the velocity in miles per second of electricity, and L the difference of longitude in time; we have

$$T + \frac{l}{V} - L = T^1 \qquad (1)$$

$$t - \frac{l}{V} - L = t^1 \qquad (2)$$

Add and we get

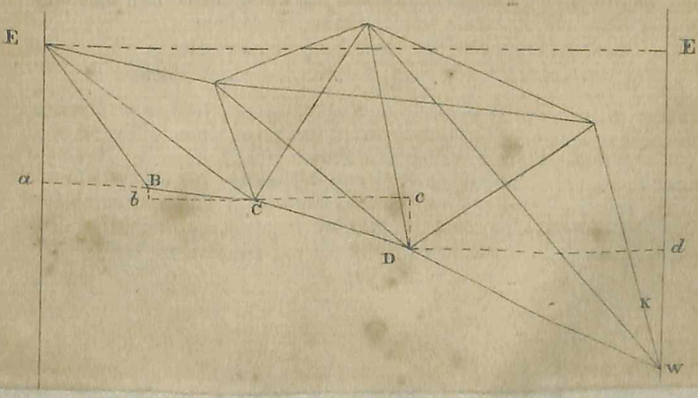
$$t + T - 2L = t^1 + T^1$$
$$2L = (T + t) - (T^1 + t^1$$
$$\text{or } L = \frac{(T + t) - (T^1 + t^1)}{2}$$

In the above method a good clock and transit instrument at each station are required; only one clock is required, if a chronograph be employed, and this clock may be any where in the circuit. The star, or stars, to be observed are agreed on before hand. When one of them becomes visible in the transit instrument of the eastern station, a signal is sent to the western and the chronographs at both stations, on which the clock marks each second, are set going. The time of transit of the star at the eastern is, therefore, recorded at both. When the transit at the western station takes place it is similarly recorded at both.

When land lines of great length are used relays are introduced, here we have a loss of time due to mechanism. Further, the time taken by the various electro-magnets to do their work, "armature time" as it is called, is believed to vary with the strength of the battery employed.

When we come to a comparison of the value of these various methods, we find the chronometric and telegraphic methods occupy the first rank, and as an example we may quote the determination of the longitude of Washington. By the astronomical method in the hands of the most distinguished astronomers, the discordance of results which appeared entitled to full reliance was about 4″. The chronometric method was brought into play, and then the result differed from that deduced by Professor Newcomb, from moon culminations extending over some years by more than 3½ seconds of time. Finally the telegraphic method was introduced by Dr. Gould and Professor Lovering, and the value obtained by transportation of chronometers was confirmed.

By Geodetical Measurements.—The difference of longitude between two stations may be obtained by connecting them by triangulation. Let E be the eastern and W the western. The two stations are then connected by triangulation, as shown in the figure below; then all the sides of the connecting triangles being known, and the azimuth of one such as E B, the distance E E′ can be calculated as follows:—



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As the length E B is known, and the angle a E B = (180 – azimuth) E a is known from the right-angled triangle a E B, also a B is known, similarly in the triangle B b C, we know the angle B C b, for it is equal (E B C + E B a) – 180°; also the length B C, and hence we can calculate b C and B b, and so on; and adding we obtain E′ E and E′ W, and from the right-angled triangle E E′ W, knowing the two sides, we can calculate the hypothenuse E W.

Now as we know E E′, it is clear that if we knew the number of feet which a degree of the small circle E E′ subtends, we would have the difference of longitude in arc by dividing E E by this number.

The length of a degree of longitude in any parallel of latitude may be calculated as follows :—

Let $l \pm 45$ be the latitude of the place, and m the length in feet of an arc which subtends 1′. Then we have

$$m = 6076{\cdot}36 \left(1 \pm \frac{\sin.\ 2\,l}{200}\right) \qquad (1)$$

Also the length in feet of an arc subtending 1′ on a great circle perpendicular to the meridian is

$$m' = 6076{\cdot}36 \left(1 + \frac{1}{300} \pm \frac{\sin.\ 2\,l}{600}\right) \qquad (2)$$

And the length of a minute of longitude is

$$m'' = m' \cos.\ \text{latitude}.$$

Hence in the formula (2), if $l \pm 45$ be the mean latitude, we can calculate the mean value of m'', and then dividing the known length E E′ by this quantity we have the value of E E′ in arc, and then by a simple reduction to time the longitude; or, if we choose, we can find the distance E W from the triangulation, and also we can calculate the azimuth of E W, let it be 180 – θ, then the number of feet subtended by one minute of the great circle, making an angle θ with meridian is $m_\theta = m \cos.^2 \theta + m' \sin.^2 \theta$; m and m' being calculated from the formulæ already given, corresponding to the mean latitude. As E W is known in feet, the number of degrees in E W becomes known, and hence we can solve for the longitude, as in the spherical Δ P E W we know the three sides, viz., the co-latitudes and E W, hence P can be found.

The longitude may also be obtained if we know the co-latitudes and azimuths of E and W, for by Napier analogies we have

$$\frac{\tan.\ \frac{1}{2}\,(E + W)}{\cot.\ \frac{1}{2}\,P} = \frac{\cos.\ \frac{1}{2}\ \text{sum co-latitudes}}{\cos.\ \frac{1}{2}\ \text{difference co-latitudes}}$$

Interpolation by Differences.—It is frequently required from a series of values corresponding to equi-distant intervals of time following any law, to deduce some intermediate time. Thus in the *Nautical Almanack*, we have the moon's R.A. and declination for every hour of the day, and its distance from certain stars, at 3 hours interval, for certain days; from this data it may be required to determine the right ascension, declination, or distance corresponding to an intermediate time, or the time corresponding to a certain R.A. declination or distance. This is effected by interpolation, *i.e.*, the intermediate position is determined from the known increments of the data corresponding to equal increments of time. The quantities on which the value of the given magnitudes depend are called arguments. Time is generally the argument in astronomical tables.

When the increments of the given quantities corresponding to equal intervals of time are constant, or when the first differences are constant, then the intermediate distance is obtained at once from the first differences by a simple proportion. This is called interpolation by first differences; but if the first differences are not constant, and the second are, then for the differences between these first differences, a correction has to be applied, and this is called correction for second differences, and so on for 3rd, 4th, or any number of differences.

The correction for second differences may be easily obtained as follows:—Let a be the quantity in the *Nautical Almanack* corresponding say to 12 hours, and supposing we want to find the value of a, corresponding to any other time, say 12h. 40m. $= t$; let D be this value of a, then if the second differences be constant, x may be expressed in the form,

$$D = a + bt + ct^2. \qquad (1)$$

Now make $t = 0$, $t = -1$, and $t = +1$.

Then we get the values of D, corresponding to 12, 11, and 1 o'clock. Let these values be D_{12}, D_{11}, and D_1; then we have

$$D_{11} = a - b + c$$
$$D_{12} = a$$
$$D_1 = a + b + c$$

Now from these equations we can find b and c by differencing as follows:—

	1st differences.	2nd differences.
$D_{12} - D_{11} =$	$b - c$	
$D_1 - D_{12} =$	$b + c$	$2\,c$

∴ Adding and subtracting we have

$$b - c + b + c = 2\,b = \text{sum 1st diff.} = \Delta_1$$
$$b + c - b + c = 2\,c = \text{sum 2nd diff.} = \Delta_2$$

$$\text{therefore } b = \frac{\Delta_1}{2}$$

$$\text{and } c = \frac{\Delta_2}{2}$$

Substituting in equation $D = a + bt + ct^2$ we have

$$D = a + \frac{\Delta_1}{2}t + \frac{\Delta_2}{2}t^2 \qquad (a)$$

Hence if $t = 40$ min., and $a =$ the R. A. or declination at 12 o'clock, the R.A. or declination at 12.40 would be

$$D = a + \frac{\Delta_1}{2}\,\frac{40}{60} + \frac{\Delta_2}{2}\left(\frac{40}{60}\right)^2$$

If the distance of the moon from a certain star were required at say 12.40, then a being the distance at 12, we would have

$$D = a + \frac{\Delta_1}{2}\left(\frac{40}{180}\right) + \frac{\Delta_2}{2}\left(\frac{40}{180}\right)$$

the distances being given for every 3 hours only.

Conversely, if we know the distance, or D, and we want to find the time, we would be able to find t by solving the quadratic equation (a), or more simply and with any degree of approximation, as follows :—

$$D = a + \frac{\Delta_1}{2}\,t + \frac{\Delta_2}{2}\,t^2$$

Now if the intervals are sufficiently near, Δ_2 is a small quantity, and hence $\frac{\Delta_2}{2}\,t^2$ may be neglected, and we get as a 1st approximate to t,

$$D = a + \frac{\Delta_1}{2}\,t$$

$$\text{therefore } t = \frac{2\,(D - a)}{\Delta_1} = \frac{2\,d}{\Delta_1}$$

Substitute this value of t in the third term of the above equation, and we have for a second approximate value of t,

$$\begin{aligned} D &= a + \frac{\Delta_1 t_1}{2} + \frac{\Delta_2}{2},\ \frac{4\,d^2}{\Delta_1} \\ &= a + \frac{\Delta_1 t_1}{2} + \frac{2\,\Delta_2\,d^2}{\Delta_1} \end{aligned}$$

or $t_1 = d - \frac{2\,\Delta_2}{\Delta_1}\,d^2$, which would give us a sufficiently close approximate for the time in most cases ; a third approximation might be found in the same way, and so on.

If the third differences are supposed constant, we would have

$$D = a + bt + ct^2 + dt^2$$

and proceeding just as above we would find

$d = \frac{1}{12}$ sum 3rd diff. $= \frac{1}{12}\,\Delta_3$.
$c = \frac{1}{6}$ sum 2nd diff. $= \frac{1}{6}\,\Delta_2$
$b = \frac{1}{3}\,\Delta_1 - \frac{1}{3}\,\Delta_3$ and hence substituting we would have

$$D = a + \left(\frac{\Delta_1}{4} - \frac{\Delta_3}{3}\right)t + \frac{\Delta_2}{6}\,t^2 + \frac{\Delta_3}{12}\,t^3$$

It is seldom necessary to push the approximation further than the third order of differences, but when this is required we may use any of the general formulæ of interpolation given in works on astronomy.

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